

I realize that, no matter how careful I have endeavored to be, occasional errors may still exist. I should be grateful if you would be kind enough to notify me as you discover them either in the book or in this manual.

Sincerely,



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A note to instructors using FIELD AND WAVE ELECTROMAGNETICS.

Dear Colleague:

As teachers of introductory electromagnetics, we are all aware of two facts: that most students consider the subject matter difficult, and that there are numerous books on the market dealing with this subject. It is understandable that students find electromagnetics difficult. First of all, the subject matter is built upon abstract models that demand a good mathematical background. Second, before the course on electromagnetics, students who have studied circuit theory normally encounter functions of only one independent variable, namely, time; whereas in electromagnetics they are suddenly required to deal with functions of four variables (space and time). This is a big transition, and visualization problems associated with solid geometry add to the difficulty. Finally, students are often confused about the way the subject matter is developed, even after they have completed the course, mainly because most books do not provide a unified and comprehensible approach.

As I point out in the Preface of the book, the inductive approach of beginning with the various experimental laws tends to be fragmented and lacks cohesiveness, whereas the practice of writing the four general Maxwell's equations at the outset without discussing their necessity and sufficiency presents a major stumbling block for learning. Students are often puzzled about the structure of the electromagnetic model. I sincerely believe that the gradual axiomatic approach based on Helmholtz's theorem used in this book provides unity in the gradational development of the electromagnetic model from the very simple model for electrostatics. Although a rigorous mathematical proof of Helmholtz's theorem is relatively involved (not included in the book), the physical concept of specifying both the flow source and the vortex (circulation) source in order to define a vector field is quite simple.

Many review questions are provided at the end of each chapter. They are designed to review and reinforce the essential material in the chapter without the need for a calculator. You may wish to use them as a vehicle for discussion in class.

I have tried to make the problems in each chapter meaningful and to avoid trivial number-plugging types. This solutions manual gives the solutions and answers to all the problems in the book. I hope it proves to be a useful aid in teaching from the book. Answers to odd-numbered problems are included in the back of the book.

Chapter 2

P.2-1 (a) $\bar{a}_1 = \frac{\bar{A}}{A} = \frac{\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 3}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{\sqrt{14}} (\bar{a}_x + \bar{a}_y 2 - \bar{a}_z 3)$

(b) $|\bar{A} - \bar{B}| = |\bar{a}_x + \bar{a}_y 6 - \bar{a}_z 4| = \sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

(c) $\bar{A} \cdot \bar{B} = 0 + 2(-4) + (-3) = -11$

(d) $\theta_{AB} = \cos^{-1}(\bar{A} \cdot \bar{B} / AB) = \cos^{-1}(-11 / \sqrt{14} \sqrt{17}) = 135.5^\circ$

(e) $\bar{A} \cdot \bar{a}_C = \bar{A} \cdot \frac{\bar{C}}{C} = \bar{A} \cdot \frac{1}{\sqrt{29}} (\bar{a}_x 5 - \bar{a}_z 2) = \frac{11}{\sqrt{29}}$

(f) $\bar{A} \times \bar{C} = -\bar{a}_x 4 - \bar{a}_y 13 - \bar{a}_z 10$

(g) $\bar{A} \cdot (\bar{B} \times \bar{C}) = (\bar{A} \times \bar{B}) \cdot \bar{C} = -42$

(h) $(\bar{A} \times \bar{B}) \times \bar{C} = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{A}(\bar{C} \cdot \bar{B}) = \bar{a}_x 2 - \bar{a}_y 40 + \bar{a}_z 5$

$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B}) = \bar{a}_x 35 + \bar{a}_y 44 - \bar{a}_z 11$

P.2-2 Position vectors of the three corners:

$\overline{OP}_1 = \bar{a}_y - \bar{a}_z 2, \quad \overline{OP}_2 = \bar{a}_x 4 - \bar{a}_y - \bar{a}_z 3, \quad \overline{OP}_3 = \bar{a}_x 6 + \bar{a}_y 2 + \bar{a}_z 5$

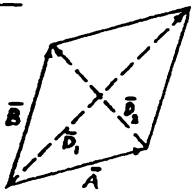
Vectors representing the three sides of the triangle:

$\overline{P_1 P_2} = \overline{OP}_2 - \overline{OP}_1 = \bar{a}_x 4 - \bar{a}_z, \quad \overline{P_2 P_3} = \bar{a}_x 2 + \bar{a}_y + \bar{a}_z 8, \quad \overline{P_3 P_1} = -\bar{a}_x 6 - \bar{a}_y - \bar{a}_z$

(a) $\overline{P_1 P_2} \cdot \overline{P_2 P_3} = 0. \quad \therefore \Delta P_1 P_2 P_3$ is a right triangle.

(b) Area of triangle = $\frac{1}{2} |\overline{P_1 P_2} \times \overline{P_2 P_3}| = 17.1$

P.2-3



$\bar{D}_1 = \bar{B} + \bar{A}, \quad \bar{D}_2 = \bar{B} - \bar{A}$

$\bar{D}_1 \cdot \bar{D}_2 = (\bar{B} + \bar{A}) \cdot (\bar{B} - \bar{A})$

$= \bar{B} \cdot \bar{B} - \bar{A} \cdot \bar{A} = 0$

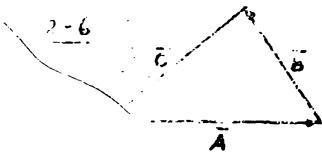
$\therefore \bar{D}_1 \perp \bar{D}_2$

P.2-4 From $\bar{A} \cdot \bar{B} = \bar{A} \cdot \bar{C}$, we have $\bar{A} \cdot (\bar{B} - \bar{C}) = 0. \quad (1)$

From $\bar{A} \times \bar{B} = \bar{A} \times \bar{C}$, we have $\bar{A} \times (\bar{B} - \bar{C}) = 0. \quad (2)$

(1) implies $\bar{A} \perp (\bar{B} - \bar{C})$ and (2) implies $\bar{A} \parallel (\bar{B} - \bar{C})$. Since \bar{A} is not a null vector, (1) and (2) cannot hold at the same time unless $(\bar{B} - \bar{C})$ is a null vector. Thus, $\bar{B} - \bar{C} = 0$ or $\bar{B} = \bar{C}$.

P.2-5 $\bar{a}_\alpha \cdot \bar{a}_\beta = \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$



$$\vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{A} \times \vec{B} = \vec{C} = \vec{A}$$

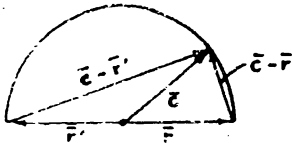
$$\vec{C} \times \vec{A} = \vec{B} = \vec{C}$$

$$\vec{B} \times \vec{C} = \vec{A} = \vec{B}$$

Magnitude relations: $AB \sin \theta_{AB} = CA \sin \theta_{CA} = BC \sin \theta_{BC}$

Hence
$$\frac{A}{\sin \theta_{BC}} = \frac{B}{\sin \theta_{CA}} = \frac{C}{\sin \theta_{AB}}$$

2-7



$$\vec{r}' = -\vec{r}, \quad r' = r$$

$$(\vec{c} - \vec{r}') \cdot (\vec{c} - \vec{r}') = (\vec{c} + \vec{r}) \cdot (\vec{c} - \vec{r})$$

$$= 0$$

$$\therefore (\vec{c} - \vec{r}') \perp (\vec{c} - \vec{r})$$

2-9 Expand $\vec{A} \times (\vec{A} \times \vec{X}) = \vec{A}(\vec{A} \cdot \vec{X}) - \vec{X}(\vec{A} \cdot \vec{A})$

or $\vec{A} \times \vec{P} = \rho \vec{A} - A^2 \vec{X}$

$$\therefore \vec{X} = \frac{1}{A^2} (\rho \vec{A} + \vec{P} \times \vec{A})$$

2-10 $\vec{A}_{P_1} = -\vec{a}_y 3 - \vec{a}_z 2, \quad \vec{O}\vec{P}_1 = -\vec{a}_y 2 + \vec{a}_z 3$

$$\vec{O}\vec{P}_2 = \vec{a}_x (r \cos \phi) + \vec{a}_y (r \sin \phi) + \vec{a}_z = \vec{a}_x \frac{\sqrt{2}}{2} - \vec{a}_y \frac{3}{2} + \vec{a}_z$$

$$\vec{P}_1 \vec{P}_2 = \vec{O}\vec{P}_2 - \vec{O}\vec{P}_1 = \vec{a}_x \frac{\sqrt{2}}{2} + \vec{a}_y \frac{1}{2} - \vec{a}_z 2, \quad |\vec{P}_1 \vec{P}_2| = \sqrt{5}$$

$$\vec{A}_{P_1} \cdot \vec{a}_{P_1 P_2} = \vec{A}_{P_1} \cdot \frac{\vec{P}_1 \vec{P}_2}{|\vec{P}_1 \vec{P}_2|} = \frac{\sqrt{5}}{2} = 1.12$$

2-11 (a) $x = r \cos \phi = 4 \cos(2\pi/3) = -2$

$$y = r \sin \phi = 4 \sin(2\pi/3) = 2\sqrt{3}$$

$$z = 3$$

(b) $R = (r^2 + z^2)^{1/2} = (4^2 + 3^2)^{1/2} = 5$

$$\theta = \tan^{-1}(r/z) = \tan^{-1}(4/3) = 53.1^\circ$$

$$\phi = 2\pi/3 = 120^\circ$$

2-12 (a) $\vec{E}_p = \vec{a}_R \frac{25}{(-3)^2 + 4^2 + (-5)^2} = \vec{a}_R \frac{1}{2}$

$$(E_p)_x = \frac{1}{2} \left(-\frac{3}{\sqrt{50}} \right) = 0.212$$

(b) $\vec{a}_R = \frac{1}{\sqrt{50}} (-\vec{a}_x 3 + \vec{a}_y 4 - \vec{a}_z 5), \quad \vec{a}_B = \frac{\vec{B}}{B} = \frac{1}{3} (\vec{a}_x 2 - \vec{a}_y 2 + \vec{a}_z)$

$$\theta = \cos^{-1}(\vec{a}_R \cdot \vec{a}_B) = \cos^{-1} \left(-\frac{19}{3\sqrt{50}} \right) = 154^\circ$$

P. 2-13 $\bar{a}_x = \bar{a}_z \sin \theta \cos \phi + \bar{a}_y \sin \theta \sin \phi + \bar{a}_z \cos \theta = \frac{\bar{a}_z x + \bar{a}_y y + \bar{a}_z z}{\sqrt{x^2 + y^2 + z^2}}$
 $\bar{a}_y = \bar{a}_z \cos \theta \cos \phi + \bar{a}_y \cos \theta \sin \phi - \bar{a}_z \sin \theta = \frac{\bar{a}_z y z + \bar{a}_y z - \bar{a}_z (x^2 + y^2)}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}}$
 $\bar{a}_\phi = -\bar{a}_x \sin \phi + \bar{a}_y \cos \phi = \frac{-\bar{a}_x y + \bar{a}_y x}{\sqrt{x^2 + y^2}}$

P. 2-14 $\int_p^q \bar{E} \cdot d\bar{l} = \int_p^q (y dx + x dy)$.

(a) $x = 2y^2, dx = 4y dy; \int_p^q \bar{E} \cdot d\bar{l} = \int_1^2 (4y^2 dy + 2y^2 dy) = 14$

(b) $x = 6y - 4, dx = 6 dy; \int_p^q \bar{E} \cdot d\bar{l} = \int_1^2 [6y dy + (6y - 4)] dy = 14$

Equal line integrals along two specific paths do not necessarily imply a conservative field. \bar{E} is a conservative field in this case because $\bar{E} = \nabla(xy + C)$.

P. 2-15 $\begin{bmatrix} E_r \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} r \sin \phi \\ r \cos \phi \end{bmatrix}$

$\bar{E} = \bar{a}_r r \sin 2\phi + \bar{a}_\phi r \cos 2\phi$

$\bar{E} \cdot d\bar{l} = r \sin 2\phi dr + r^2 \cos 2\phi d\phi$

$P_3(3, 4, -1) = P_3(5, 53.1^\circ, -1); P_4(4, -3, -1) = P_4(5, -36.9^\circ, -1)$

There is no change in r ($= 5$) from P_3 to P_4 .

$\therefore \int_{P_3}^{P_4} \bar{E} \cdot d\bar{l} = 5^2 \int_{53.1^\circ}^{-36.9^\circ} \cos 2\phi d\phi = -24$

P. 2-16 (a) $\nabla V = \left[\bar{a}_x \left(\frac{\pi}{2} \cos \frac{\pi}{2} x \right) \left(\sin \frac{\pi}{3} y \right) + \bar{a}_y \left(\sin \frac{\pi}{2} x \right) \left(\frac{\pi}{3} \cos \frac{\pi}{3} y \right) + \bar{a}_z \left(\sin \frac{\pi}{2} x \right) \left(\sin \frac{\pi}{3} y \right) \right] e^{-z}$

$(\nabla V)_\rho = -\left(\bar{a}_y \frac{\pi}{6} + \bar{a}_z \frac{\sqrt{3}}{2} \right) e^{-3} = -\left(\bar{a}_y 0.026 + \bar{a}_z 0.043 \right)$

(b) $\bar{\rho} \cdot \bar{0} = -\bar{a}_x - \bar{a}_y 2 - \bar{a}_z 3; \bar{a}_{\rho 0} = -\frac{1}{\sqrt{14}} (\bar{a}_x + \bar{a}_y 2 + \bar{a}_z 3)$

$\therefore (\nabla V)_\rho \cdot \bar{a}_{\rho 0} = \frac{1}{\sqrt{14}} \left(\frac{\pi}{3} - \frac{3\sqrt{3}}{2} \right) e^{-3} = 0.0485$

P. 2-17 $\oint (\bar{a}_r 3 \sin \theta) \cdot (\bar{a}_r 5^2 \sin \theta) d\theta d\phi = \int_0^{2\pi} \int_0^\pi 75 \sin^2 \theta d\theta d\phi = 75\pi^2$

P. 2-19 $\oint \bar{A} \cdot d\bar{s} = \left(\int_{\text{top face}} + \int_{\text{bottom face}} + \int_{\text{walls}} \right) \bar{A} \cdot d\bar{s}$

Top face ($z=4$): $\bar{A} = \bar{a}_r r^2 + \bar{a}_z 8$, $d\bar{s} = \bar{a}_z ds$.

$$\int_{\text{top face}} \bar{A} \cdot d\bar{s} = \int_{\text{top face}} 8 ds = 8(\pi 5^2) = 200\pi.$$

Bottom face ($z=0$): $\bar{A} = \bar{a}_r r^2$, $d\bar{s} = -\bar{a}_z ds$.

$$\int_{\text{bottom face}} \bar{A} \cdot d\bar{s} = 0.$$

Walls ($r=5$): $\bar{A} = \bar{a}_r 25 + \bar{a}_z 2z$, $ds = \bar{a}_r ds$.

$$\int_{\text{wall}} \bar{A} \cdot d\bar{s} = 25 \int_{\text{wall}} ds = 25(2\pi 5 \times 4) = 1000\pi.$$

$$\therefore \oint \bar{A} \cdot d\bar{s} = 200\pi + 0 + 1000\pi = 1,200\pi.$$

$$\bar{\nabla} \cdot \bar{A} = 3r + 2, \quad \int \bar{\nabla} \cdot \bar{A} dv = \int_0^4 \int_0^{2\pi} \int_0^5 \bar{\nabla} \cdot \bar{A} r dr d\phi dz = 1,200\pi.$$

P.2-20 $\bar{\nabla} \cdot \bar{F} = \frac{1}{r} \frac{\partial}{\partial r} (rF_r) + \frac{\partial}{\partial z} F_z = k_2$, $\int \bar{\nabla} \cdot \bar{F} dv = 24\pi k_2$.

Divergence theorem fails because \bar{F} has a singularity at $r=0$.

P.2-21 a) $\oint \bar{D} \cdot d\bar{s} = \oint \left(\frac{\cos^2 \phi}{R^3} \right) R^2 \sin \theta d\theta d\phi$

$$= \int_0^{2\pi} \int_0^\pi \left(\frac{1}{2} - 1 \right) \cos^2 \phi \sin \theta d\theta d\phi = -\pi.$$

b) $\bar{\nabla} \cdot \bar{D} = -(\cos^2 \phi)/R^4$, $\int \bar{\nabla} \cdot \bar{D} dv = \int_0^{2\pi} \int_0^\pi \int_0^1 (\bar{\nabla} \cdot \bar{D}) R^2 \sin \theta dr d\theta d\phi = -\pi$.

P.2-22 $\bar{\nabla} \cdot \bar{F} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 F_R) = \frac{1}{R^2} \frac{\partial}{\partial R} [R^2 f(R)] = 0$.

$$\therefore R^2 f(R) = \text{constant}, C; \text{ i.e., } f(R) = \frac{C}{R^2}.$$

P.2-24 a) $\bar{A} \cdot d\bar{l} = 3x^2 y^2 dx - x^2 y^2 dy$

$$\oint \bar{A} \cdot d\bar{l} = 21 + \frac{56}{3} - 7 = \frac{98}{3} = 32\frac{2}{3}.$$

b) $\bar{\nabla} \times \bar{A} = -\bar{a}_z (2x^2 y^2)$

$$\int (\bar{\nabla} \times \bar{A}) \cdot d\bar{s} = \int_1^2 \int_y^2 (-\bar{a}_z (2x^2 y^2)) \cdot (-\bar{a}_z dx dy) = 32\frac{2}{3}.$$

c) No, because $\bar{\nabla} \times \bar{A} \neq 0$.

P.2-25 $\bar{\nabla} \times \bar{A} = \frac{1}{R \sin \theta} (\bar{a}_z \cos \theta \sin \frac{\phi}{2} - \bar{a}_\theta \sin \theta \sin \frac{\phi}{2})$

$$\int_S (\bar{\nabla} \times \bar{A}) \cdot d\bar{s} = \int_0^{2\pi} \int_0^{\pi/2} (\bar{\nabla} \times \bar{A})_{z=b} \cdot (\bar{a}_z b^2 \sin \theta d\theta d\phi) = 4b.$$

$$\oint_C \vec{A} \cdot d\vec{L} = \int_0^{2\pi} (\vec{A})_{\theta=b} \cdot (\vec{a}_\theta b d\phi) = \int_0^{2\pi} b \sin \frac{\phi}{2} d\phi = 4b.$$

Chapter 3

P.3-1 a) $\alpha = \tan^{-1} \left(\frac{L-w}{d_1} \right) = \tan^{-1} \left(\frac{m v_0^2}{e w E_d} \right).$

b) $d_1 = \frac{d_0}{20}, \quad \frac{e E_d}{2m} \frac{w^2}{v_0^2} = \frac{1}{20} \frac{e E_d}{m v_0^2} w(L - \frac{w}{2}), \quad \frac{L}{w} = 10.5$

P.3-2 a) Max. voltage V_{max} will make $d_1 = h/2$ at $z=w$.

$$\frac{h}{2} = \frac{e}{2m} \left(\frac{V_{max}}{h} \right) \left(\frac{w}{v_0} \right)^2, \quad \text{or } V_{max} = \frac{m}{e} \left(\frac{v_0 h}{w} \right)^2.$$

b) At the screen, $(d_0)_{max} = D/2$. Hence L must be $\leq L_{max}$ where

$$L_{max} = \frac{1}{2} \left(w + \frac{m v_0^2 D h}{e w V_{max}} \right).$$

c) Double V_{max} by doubling v_0^2 , or doubling the anode accelerating voltage.

P.3-3 $F = \frac{e^2}{4\pi\epsilon_0 R^2} = (9 \times 10^9) \frac{(1.602 \times 10^{-19})^2}{(5.28 \times 10^{-11})^2} = 8.29 \times 10^{-8} \text{ (N)}.$
Attractive force.

P.3-4 $\vec{Q}_1 \cdot \vec{p} = -\bar{a}_x 2 - \bar{a}_y; \quad \vec{Q}_2 \cdot \vec{p} = -\bar{a}_x 3 + \bar{a}_y.$

$$\vec{E}_{p1} = \frac{Q_1}{4\pi\epsilon_0 (\sqrt{5})^3} (-\bar{a}_x 2 - \bar{a}_y); \quad \vec{E}_{p2} = \frac{Q_2}{4\pi\epsilon_0 (\sqrt{10})^3} (-\bar{a}_x 3 + \bar{a}_y)$$

a) No x -component of \vec{E}_p : $-\frac{2Q_1}{(\sqrt{5})^3} - \frac{3Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = -\frac{3}{4\sqrt{2}}$

b) No y -component of \vec{E}_p : $-\frac{Q_1}{(\sqrt{5})^3} + \frac{Q_2}{(\sqrt{10})^3} = 0$, or $\frac{Q_1}{Q_2} = \frac{1}{2\sqrt{2}}$

P.3-5

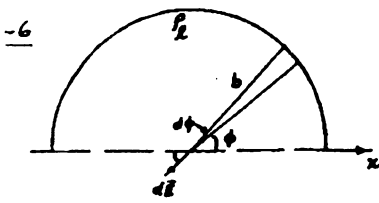


At equilibrium, electric force \vec{F}_e and gravitational force \vec{F}_m must add to give a resultant along the thread.

$$\frac{F_e}{F_m} = \tan 5^\circ = 0.0875.$$

$$F_m = mg = 9.80 \times 10^{-4} \text{ (N)}$$

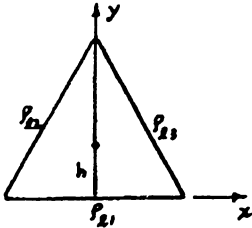
$$F_e = \frac{Q^2}{4\pi\epsilon_0 (2 \times a_2 \sin 5^\circ)^2} = 7.41 \times 10^{-12} Q^2 \text{ (N)}. \quad Q = 3.40 \text{ ()}$$



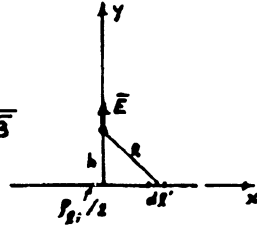
$$dE_y = -\frac{\rho_L (b d\phi)}{4\pi\epsilon_0 b^2} \sin\phi,$$

$$\begin{aligned} \vec{E} &= \vec{a}_y E_y = -\vec{a}_y \frac{\rho_L}{4\pi\epsilon_0 b} \int_0^\pi \sin\phi d\phi \\ &= -\vec{a}_y \frac{\rho_L}{2\pi\epsilon_0 b}. \end{aligned}$$

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$$h = \frac{L}{2\sqrt{3}}$$



\vec{E} at the center of triangle would be zero if all three line charges were of the same charge density. The problem is equivalent to that of a single line charge of density $\rho_L/2$. By symmetry, there will only be a y -component.

$$\begin{aligned} \vec{E} &= \vec{a}_y E_y = \vec{a}_y \int_{-L/2}^{L/2} \frac{(\rho_L/2) dl' (h)}{4\pi\epsilon_0 R^2} \left(\frac{h}{R}\right) = \vec{a}_y \int_{-L/2}^{L/2} \frac{\rho_L h dl'}{8\pi\epsilon_0 (h^2 + l'^2)^{3/2}} \\ &= \vec{a}_y \frac{3\rho_L}{4\pi\epsilon_0 L} = \vec{a}_y \frac{3\rho_L}{2\pi\epsilon_0 L}. \end{aligned}$$

1-8 Use Gauss's law: $\oint \vec{E} \cdot d\vec{s} = Q/\epsilon_0$.

a) \vec{E} is normal to the two faces at $x = \pm 0.05$ (m), where $\vec{E} = \pm \vec{a}_x E$ and $\vec{a}_n = \pm \vec{a}_x$ respectively.

$$Q = 2\epsilon_0 (5 \times 0.1^2) = 0.1\epsilon_0 = 8.84 \times 10^{-12} \text{ (C)}.$$

b) $\vec{E} = \vec{a}_r (100x) \cos\phi - \vec{a}_\phi (100x) \sin\phi = \vec{a}_r (100r \cos^2\phi) - \vec{a}_\phi (100r \sin^2\phi)$.

$$\oint \vec{E} \cdot \vec{a}_n ds = \int_0^{2\pi} (100 \times 0.05 \cos^2\phi) (0.1 \times 0.05 d\phi) = 0.025\pi.$$

$$Q = 0.0785\epsilon_0 = 6.94 \times 10^{-13} \text{ (C)}.$$

3-9 Spherical symmetry: $\vec{E} = \vec{a}_r E_r$. Apply Gauss's law.

$$1) 0 \leq R \leq b. \quad 4\pi R^2 E_r = \frac{\rho_0}{\epsilon_0} \int_0^R (1 - \frac{R'^2}{b^2}) 4\pi R'^2 dR' = \frac{4\pi\rho_0}{\epsilon_0} \left(\frac{R^3}{3} - \frac{R^5}{5b^2}\right).$$

$$E_{r1} = \frac{\rho_0}{\epsilon_0} R \left(\frac{1}{3} - \frac{R^2}{5b^2}\right).$$

$$2) b \in R < R_i: 4\pi R^2 E_{Ri} = \frac{\rho}{\epsilon_0} \int_0^b \left(1 - \frac{R^2}{b^2}\right) 4\pi R^2 dR = \frac{8\pi\rho}{15\epsilon_0} b^3$$

$$E_{Ri} = \frac{2\rho b^3}{15\epsilon_0 R^2}$$

$$3) R_i \leq R \leq R_o: E_{Ri} = 0$$

$$4) R > R_o: E_{Ra} = \frac{2\rho_o b^3}{15\epsilon_0 R^2}$$

P.3-10 Cylindrical symmetry: $\vec{E} = \vec{a}_r E_r$. Apply Gauss's law.

$$a) r < a, E_r = 0; \quad a < r < b, E_r = a\rho_{sa}/\epsilon_0 r;$$

$$r > b, E_r = (a\rho_{sa} + b\rho_{sb})/\epsilon_0 r.$$

$$b) b/a = -\rho_{sa}/\rho_{sb}$$

P.3-11 Refer to Eq. (3-49) and Fig. 3-14. \vec{E} will have no z-component if $E_R \cos\theta = E_\theta \sin\theta$, or $2 \cos^2\theta = \sin^2\theta$
 $\theta = 54.7^\circ$ and 125.3° .

P.3-12

$$P(R, \theta, \phi) \quad V = \frac{\rho}{4\pi\epsilon_0 R} \left(\frac{R}{R_1} + \frac{R}{R_2} - 2 \right)$$

$$R_1^2 = R^2 + \left(\frac{d}{2}\right)^2 - R d \cos\theta$$

$$\frac{R}{R_1} = \left[1 + \left(\frac{d}{2R}\right)^2 - \frac{d}{R} \cos\theta \right]^{-1/2}$$

$$\approx 1 + \frac{d}{2R} \cos\theta + \frac{d^2}{4R^2} \frac{3\cos^2\theta - 1}{2}$$

$$\frac{R}{R_2} \approx 1 - \frac{d}{2R} \cos\theta + \frac{d^2}{4R^2} \frac{3\cos^2\theta - 1}{2}$$

$$a) \therefore V = \frac{\rho(d/2)^2}{4\pi\epsilon_0 R} (3\cos^2\theta - 1), \quad R^2 \gg d^2$$

$$\vec{E} = -\nabla V = -\vec{a}_R \frac{\partial V}{\partial R} - \vec{a}_\theta \frac{\partial V}{\partial \theta} = \frac{\rho(d/2)^2}{4\pi\epsilon_0 R^2} \left[\vec{a}_R 3(3\cos^2\theta - 1) + \vec{a}_\theta 3 \sin 2\theta \right]$$

b) Equation for equipotential surfaces:

$$R = C_v (3\cos^2\theta - 1)^{1/3}$$

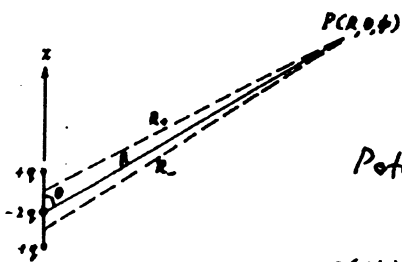
Equation for streamlines:

$$\frac{dR}{E_R} = \frac{R d\theta}{E_\theta}, \quad \text{or} \quad \frac{dR}{3\cos^2\theta - 1} = \frac{R d\theta}{\sin 2\theta}$$

$$\frac{dR}{R} = \frac{3 d(\sin\theta)}{2 \sin\theta} - \frac{d\theta}{\sin 2\theta}$$

$$R = C_s \left(\sin^{3/2}\theta / \sqrt{|\tan\theta|} \right) \quad (\text{Also see next})$$

- 12 A simpler approach (not to the same degree of approximation) is to consider the problem as a pair of displaced dipoles, each with a moment $\bar{p} = q\bar{d}/2$.



From Eq. (3-48), potential due to \bar{p} is

$$V_+ = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_+^2}$$

Potential due to $-\bar{p}$ is

$$V_- = -\frac{q(d/2)\cos\theta}{4\pi\epsilon_0 R_-^2}$$

$$V = V_+ + V_- = \frac{q(d/2)\cos\theta}{4\pi\epsilon_0} \left(\frac{1}{R_+^2} - \frac{1}{R_-^2} \right)$$

$$R_+^{-2} \cong R^{-2} \left(1 + \frac{d}{2R} \cos\theta \right), \quad R_-^{-2} \cong R^{-2} \left(1 - \frac{d}{2R} \cos\theta \right)$$

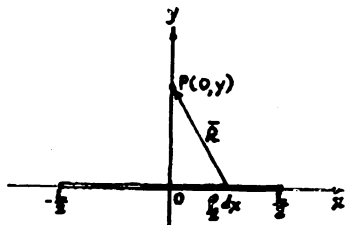
a) $\therefore V = \frac{q(d\cos\theta)^2/2}{4\pi\epsilon_0 R^3}$

$$\bar{E} = -\bar{\nabla}V = \frac{q d^2/2}{4\pi\epsilon_0 R^3} (\bar{a}_r 3\cos^2\theta + \bar{a}_\theta \sin 2\theta)$$

b) Equation for equipotential surfaces: $R = C_1' (\cos\theta)^{2/3}$

Equation for streamlines: $R = C_2' (\sin\theta)^{2/3}$

13



$$\begin{aligned} \text{a) } V &= 2 \int_0^{L/2} \frac{\rho_L dx}{4\pi\epsilon_0 R} \\ &= \frac{\rho_L}{2\pi\epsilon_0} \int_0^{L/2} \frac{dx}{\sqrt{x^2 + y^2}} \\ &= \frac{\rho_L}{2\pi\epsilon_0} \left\{ \ln \left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2} - \frac{L}{2} \right] - \ln y \right\} \end{aligned}$$

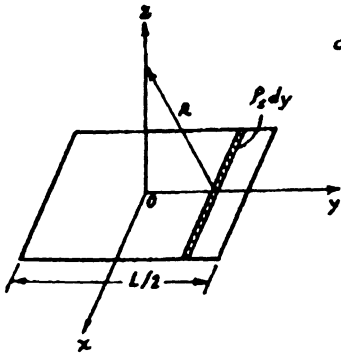
b) From Coulomb's law:

$$\bar{E} = \bar{a}_y E_y = 2 \int_0^{L/2} \bar{a}_y \frac{\rho_L y dx}{4\pi\epsilon_0 R^3} = \bar{a}_y \frac{\rho_L}{2\pi\epsilon_0 y} \frac{L/2}{\sqrt{(L/2)^2 + y^2}}$$

c) $\bar{E} = -\bar{\nabla}V$ gives the same answer.

14 Surface charge density $\rho_s = \frac{Q}{L^2}$.

Use the results of problem P. 3-13 for the coordinate system chosen in the figure on the next page. Replace ρ_L by $\rho_s dy$ and y by $\sqrt{y^2 + x^2}$.



$$\begin{aligned}
 \text{a) } V &= 2 \cdot \frac{\rho_s}{2\pi\epsilon_0} \int_0^{L/2} \left\{ \ln \left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2} + z + \left(\frac{L}{2}\right) \right] \right. \\
 &\quad \left. - \ln \sqrt{y^2 + z^2} \right\} dy \\
 &= \frac{\rho_s}{\pi\epsilon_0 L^2} \left\{ \frac{1}{2} \ln \left[\frac{\sqrt{2\left(\frac{L}{2}\right)^2 + z^2} + \frac{L}{2}}{\sqrt{2\left(\frac{L}{2}\right)^2 + z^2} - \frac{L}{2}} \right] \right. \\
 &\quad \left. - z \tan^{-1} \left[\frac{\left(\frac{L}{2}\right)^2}{z \sqrt{2\left(\frac{L}{2}\right)^2 + z^2}} \right] \right\}.
 \end{aligned}$$

$$\text{b) } \vec{E} = -\vec{\nabla} V = \frac{\rho_s Q}{\pi\epsilon_0 L^2} \tan^{-1} \left[\frac{\left(\frac{L}{2}\right)^2}{z \sqrt{2\left(\frac{L}{2}\right)^2 + z^2}} \right].$$

P.3-15 Assume the circular tube sits on the xy -plane with its axis coinciding with the z -axis. The surface charge on the tube wall is $\rho_s = Q/2\pi bh$. First find the potential along the axis at z due to a circular line charge of density ρ_s' situated at z' .

$$V = \oint \frac{\rho_s' dl}{4\pi\epsilon_0 R} = \int_0^{2\pi} \frac{\rho_s' b d\phi}{4\pi\epsilon_0 \sqrt{b^2 + (z-z')^2}} = \frac{\rho_s' b}{2\epsilon_0 \sqrt{b^2 + (z-z')^2}}.$$

a) The expression above is the contribution dV due to a circular line charge of density $\rho_s' = \rho_s dz'$.

$$dV = \frac{\rho_s' b dz'}{2\epsilon_0 \sqrt{b^2 + (z-z')^2}}.$$

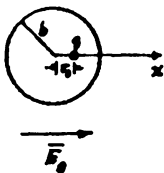
At a point outside the tube:

$$V = \int_{z'=0}^{z'=h} dV = \frac{b\rho_s}{2\epsilon_0} \ln \frac{z + \sqrt{b^2 + z^2}}{(z-h) + \sqrt{b^2 + (z-h)^2}}.$$

$$\vec{E} = -\vec{a}_z \frac{dV}{dz} = -\vec{a}_z \frac{b\rho_s}{2\epsilon_0} \left[\frac{1}{\sqrt{b^2 + (z-h)^2}} - \frac{1}{\sqrt{b^2 + z^2}} \right].$$

b) Same expressions are obtained for V and \vec{E} at a point inside the tube.

3-16

Applied \vec{E}_0 causes a displacement r_0 .Force of separation: qE_0 ;Restoring force (attraction): qE_x . E_x at q due to spherical volume of electrons of radius r_0 is (byGauss's law) $E_x = \frac{\rho r_0}{3\epsilon_0} = -\frac{\rho}{3\epsilon_0} |r_0|$

$$|r_0| = \frac{q}{\frac{4}{3}\pi b^3} = \frac{3N|e|}{4\pi b^3}$$

At equilibrium: $E_0 = |E_x| = \frac{\rho N|e|}{4\pi\epsilon_0 b^3}$, or $r_0 = \frac{4\pi\epsilon_0 b^3}{N|e|} E_0$.

3-17 $W = -q \int \vec{E} \cdot d\vec{r} = -q \int (y dx + x dy)$.

a) Along the parabola $x = 2y^2$; $dx = 4y dy$

$$W = -q \int_0^2 6y^2 dy = -14q = 28 \mu\text{J}.$$

b) Along the straight line $x = 6y - 4$; $dx = 6 dy$

$$W = -q \int_0^2 (12y - 4) dy = -14q = 28 \mu\text{J}.$$

3-18 a) $\rho_{ps} = \vec{p} \cdot \vec{a}_n = p_0 \frac{1}{2}$ on all six faces of the cube.

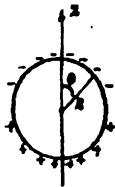
$$\rho_p = -\vec{v} \cdot \vec{p} = -3p_0.$$

$$b) Q_s = 6L^2 \rho_{ps} = 3p_0 L^3, \quad Q_v = L^3 \rho_p = -3p_0 L^3.$$

Total bound charge = $Q_s + Q_v = 0$.3-19 Assume $\vec{p} = \vec{a}_z p$. Surface charge density $\rho_{ps} = \vec{p} \cdot \vec{a}_n$

$$= (\vec{a}_z p) \cdot (\vec{a}_n)$$

$$= -p \cos \theta.$$

The z-component of the electric field intensity due to a ring of ρ_{ps} contained in width $R d\theta$ at θ is

$$dE_z = \frac{\rho_{ps} \cos \theta}{4\pi\epsilon_0 R^2} (2\pi R \sin \theta) (R d\theta) \cos \theta$$

$$= \frac{p}{2\epsilon_0} \cos^2 \theta \sin \theta d\theta.$$

At the center of the cavity: $\vec{E} = \vec{a}_z E_z = \vec{a}_z \frac{p}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta = \frac{\vec{p}}{3\epsilon_0}$.

- P.3-20 a) $V_b = E_{ba} d = 3 \times 50 = 150 \text{ (kV)}$
 b) $V_b = E_{bp} d = 20 \times 50 = 1,000 \text{ (kV)}$
 c) $V_b = E_{ba}(d-d_p) + \frac{1}{3} E_{bp} d_p = 3(40 + \frac{1}{3} \times 10) = 130 \text{ (kV)}$

P.3-21 At the $z=0$ plane : $\vec{E}_1 = \vec{a}_x 2y - \vec{a}_y 3x + \vec{a}_z 5$.

$$\vec{E}_{1t}(z=0) = \vec{E}_{2t}(z=0) = \vec{a}_x 2y - \vec{a}_y 3x$$

$$\vec{D}_{1n}(z=0) = \vec{D}_{2n}(z=0) \rightarrow 2\vec{E}_{1n}(z=0) = 3\vec{E}_{2n}(z=0) \\ \rightarrow \vec{E}_{2n}(z=0) = \frac{2}{3}(\vec{a}_z 5) = \vec{a}_z \frac{10}{3}$$

$$\therefore \vec{E}_2(z=0) = \vec{a}_x 2y - \vec{a}_y 3x + \vec{a}_z \frac{10}{3}$$

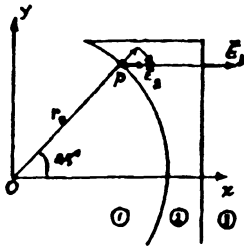
$$\vec{D}_2(z=0) = (\vec{a}_x 6y - \vec{a}_y 9x + \vec{a}_z 10)\epsilon_0$$

P.3-22 $\vec{D}_2 = \epsilon_0(\epsilon_r - 1)\vec{E}_2$: $\vec{E}_{21} = \vec{E}_{22} \rightarrow \frac{1}{\epsilon_{r1}-1} P_{21} = \frac{1}{\epsilon_{r2}-1} P_{22}$

$$\epsilon_{r1}\vec{E}_{21} = \epsilon_{r2}\vec{E}_{22} \rightarrow \frac{\epsilon_{r1}}{\epsilon_{r1}-1} P_{21} = \frac{\epsilon_{r2}}{\epsilon_{r1}-1} P_{22}$$

P.3-23 $\epsilon_1 \frac{\partial V_1}{\partial n} = \epsilon_2 \frac{\partial V_2}{\partial n}$ and $V_1 = V_2$.

P.3-24



Assume $\vec{E}_2 = \vec{a}_x E_{2r} + \vec{a}_\phi E_{2\phi}$

$$\text{B.C.: } \vec{a}_n \times \vec{E}_1 = \vec{a}_n \times \vec{E}_2 \rightarrow E_{2\phi} = -$$

For \vec{E}_3 , and hence \vec{E}_2 , to be parallel to the x -axis,

$$E_{2\phi} = -E_{2r} \rightarrow E_{2r} = 3$$

$$\text{B.C.: } \vec{a}_n \cdot \vec{D}_1 = \vec{a}_n \cdot \vec{D}_2 \rightarrow 5 = 3\epsilon_2$$

$$\therefore \epsilon_{r2} = 5/3$$

P.3-25 $\epsilon = \frac{\epsilon_2 - \epsilon_1}{d} y + \epsilon_1$

Assume Q on plate at $y=d$. $\vec{E} = -\vec{a}_y \frac{P_r}{\epsilon} = -\vec{a}_y \frac{Q}{S(\frac{\epsilon_2 - \epsilon_1}{d} y)}$

$$V = -\int_y^d \vec{E} \cdot d\vec{l} = \frac{Qd \ln(\epsilon_2/\epsilon_1)}{S(\epsilon_2 - \epsilon_1)}$$

$$C = \frac{Q}{V} = \frac{S(\epsilon_2 - \epsilon_1)}{d \ln(\epsilon_2/\epsilon_1)}$$

P.3-26 a) $C = 4\pi\epsilon_0 R = \frac{1}{9} \times 10^{-9} \times (6.37 \times 10^6)^2 = 7.08 \times 10^{-4} \text{ (F)}$

b) $E_b = 3 \times 10^6 = \frac{Q_{\max}}{4\pi\epsilon_0 R^2}$. $Q_{\max} = 1.35 \times 10^{10} \text{ (C)}$

P.3-27 Assume charge Q on conducting sphere.

$$b < R < b+d, \quad \vec{E}_1 = \vec{a}_R \frac{Q}{4\pi\epsilon_0(1+\chi_2)R^2}$$

$$R > b+d, \quad \vec{E}_2 = \vec{a}_R \frac{Q}{4\pi\epsilon_0 R^2}$$

$$V = -\int_{\infty}^b \vec{E} \cdot d\vec{R} = -\int_{\infty}^{b+d} E_2 dR - \int_{b+d}^b E_1 dR = \frac{Q}{4\pi\epsilon_0(1+\chi_2)} \left(\frac{x_0}{b+d} + \frac{1}{b} \right)$$

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0(1+\chi_2)}{\frac{x_0}{b+d} + \frac{1}{b}}$$

P.3-28 Assume charge Q on inner shell, $-Q$ on outer shell.

$$R_i < R < R_o, \quad \vec{D} = \vec{a}_R \frac{Q}{4\pi R^2}$$

$$R_i < R < b, \quad \vec{E}_1 = \frac{\vec{D}}{\epsilon_0 \epsilon_r}; \quad b < R < R_o, \quad \vec{E}_2 = \frac{\vec{D}}{2\epsilon_0 \epsilon_r}$$

$$V = -\int_{R_o}^{R_i} \vec{E} \cdot d\vec{R} = -\int_{R_o}^b E_2 dR - \int_b^{R_i} E_1 dR = \frac{Q}{4\pi\epsilon_0 \epsilon_r} \left(\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_o} \right)$$

a) $\vec{D} = \vec{a}_R \frac{\epsilon_0 \epsilon_r V}{R^2 \left(\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_o} \right)}$, $R_i < R < R_o$. $\vec{D} = 0, \vec{E} = 0$ for $R < R_i$ and $R > R_o$.

$$\vec{E}_1 = \vec{a}_R \frac{V}{R^2 \left(\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_o} \right)}; \quad \vec{E}_2 = \vec{a}_R \frac{V}{R^2 \left(\frac{1}{R_i} - \frac{1}{b} - \frac{1}{2R_o} \right)}$$

b) $C = \frac{Q}{V} = \frac{4\pi\epsilon_0 \epsilon_r}{\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_o}}$

P.3-29 Let ρ_L be the lineal charge density on the inner conductor.

$$\vec{E} = \vec{a}_r \frac{\rho_L}{2\pi\epsilon r}, \quad V_o = -\int_b^a \vec{E} \cdot d\vec{r} = \frac{\rho_L}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)$$

$$\rho_L = \frac{2\pi\epsilon V_o}{\ln(b/a)}$$

a) $\vec{E}(a) = \vec{a}_r \frac{V_o}{a \ln(b/a)}$

b) Let $x = b/a$, $f(x) = \frac{\ln x}{x}$, $\frac{\partial f(x)}{\partial x} = 0 \rightarrow \ln x = 1$,
 $x = \frac{b}{a} = 2.718$.

c) $C = \frac{\rho_L}{V_o} = \frac{2\pi\epsilon}{\ln(b/a)} = 2\pi\epsilon$ (F/m).

P.3-30 $\vec{D} = \vec{a}_r \frac{\rho_L}{2\pi r}$, $\vec{E}_1 = \vec{a}_r \frac{\rho_L}{2\pi\epsilon_0 \epsilon_r r}$, $r_i < r < b$; $\vec{E}_2 = \vec{a}_r \frac{\rho_L}{2\pi\epsilon_0 \epsilon_r r}$, $b < r < r_o$

$$V = -\int_{r_o}^{r_i} \vec{E} \cdot d\vec{r} = \frac{\rho_L}{2\pi\epsilon_0} \left[\frac{1}{\epsilon_r} \ln\left(\frac{b}{r_i}\right) + \frac{1}{\epsilon_r} \ln\left(\frac{r_o}{b}\right) \right]$$

$$C = \frac{\rho_L}{V} = \frac{2\pi\epsilon_0}{\frac{1}{\epsilon_r} \ln\left(\frac{b}{r_i}\right) + \frac{1}{\epsilon_r} \ln\left(\frac{r_o}{b}\right)} \quad (\text{F/m})$$

P.3-31 From Gauss's law, $\oint \bar{D} \cdot d\bar{s} = \rho_L L$.

$$\bar{E}_1 = \bar{E}_2 = \bar{a}_r E_r. \quad \pi r L (\epsilon_0 \epsilon_{r1} + \epsilon_0 \epsilon_{r2}) E_r = \rho_L L.$$

$$E_r = \frac{\rho_L}{\pi r \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})}; \quad V = -\int_{r_1}^{r_2} E_r dr = \frac{\rho_L}{\pi \epsilon_0 (\epsilon_{r1} + \epsilon_{r2})} \ln\left(\frac{r_2}{r_1}\right).$$

$$\therefore C = \frac{\rho_L L}{V} = \frac{\pi \epsilon_0 (\epsilon_{r1} + \epsilon_{r2}) L}{\ln(r_2/r_1)}.$$

P.3-32 $\bar{E} = \bar{a}_r \frac{\rho_L}{2\pi \epsilon r} = \bar{a}_r \frac{\rho_L}{2\pi \epsilon_0 (2 + \frac{r}{2})r} = \bar{a}_r \frac{\rho_L}{4\pi \epsilon_0 (r+2)}$

$$V = -\int_{r_1}^{r_2} \bar{E} \cdot d\bar{r} = \frac{\rho_L}{4\pi \epsilon_0} \ln(r+2) \Big|_5^7 = \frac{\rho_L}{4\pi \epsilon_0} \ln\left(\frac{9}{7}\right).$$

$$C = \frac{\rho_L L}{V} = \frac{4\pi L \epsilon_0}{\ln(9/7)} = 1500 \epsilon_0 = 13.26 \mu\text{F}.$$

P.3-33 $\bar{D} = \bar{a}_r \frac{R}{3} \rho, R < b; \quad \bar{D} = \bar{a}_r \frac{b^3 \rho}{3R^2}, R > b; \quad \bar{E} = \frac{1}{\epsilon_0} \bar{D}$.

a) $W_i = \frac{1}{2} \int_V \bar{D} \cdot \bar{E} dv = \frac{1}{2} \int_0^b \frac{1}{\epsilon_0} \left(\frac{R}{3} \rho\right)^2 4\pi R^2 dR = \frac{2\pi b^5 \rho^2}{45 \epsilon_0}$.

b) $W_o = \frac{1}{2} \int_b^\infty \frac{1}{\epsilon_0} \left(\frac{b^3 \rho}{3R^2}\right)^2 4\pi R^2 dR = \frac{2\pi b^5 \rho^2}{9 \epsilon_0}$.

$$\text{Total } W = W_i + W_o = \frac{4\pi b^5 \rho^2}{15 \epsilon_0}$$

P.3-34 $\bar{E} = \frac{\rho}{4\pi \epsilon_0 R^2} (\bar{a}_R 2 \cos \theta + \bar{a}_\theta \sin \theta)$.

$$W = \frac{1}{2} \epsilon_0 \int_V E^2 dv = \frac{\epsilon_0}{2} \left(\frac{\rho}{4\pi \epsilon_0}\right)^2 \int_0^{2\pi} d\phi \int_0^\pi d\theta \int_b^\infty \frac{1}{R^6} (4 \cos^2 \theta + \sin^2 \theta) R^2 \sin \theta dR$$

$$= \frac{\rho^2}{12\pi \epsilon_0 b^3}$$

P.3-35 Two conductors at potentials V_1 and V_2 carrying charges

+Q and -Q: $W_e = \frac{1}{2} V_1 \int_S \rho_s ds + \frac{1}{2} V_2 \int_S \rho_s ds = \frac{1}{2} Q (V_1 - V_2)$

$$= \frac{1}{2} C V^2, \quad V = V_1 - V_2.$$

P.3-36 a) Region 1 — dielectric; region 2 — air.

$$\bar{E}_1 = -\bar{a}_y \frac{V_0}{d}, \quad \bar{D}_1 = -\bar{a}_y \epsilon_0 \epsilon_r \frac{V_0}{d}, \quad \rho_{s1} = \epsilon_0 \epsilon_r \frac{V_0}{d} \text{ (top plate)}$$

$$\bar{E}_2 = -\bar{a}_y \frac{V_0}{d}, \quad \bar{D}_2 = -\bar{a}_y \epsilon_0 \frac{V_0}{d}, \quad \rho_{s2} = \epsilon_0 \frac{V_0}{d} \text{ (top plate)}$$

$$\frac{W_{e1}}{W_{e2}} = \frac{\epsilon_r x}{L-x} = 1 \rightarrow x = \frac{L}{\epsilon_r + 1}$$

P.3-37 $\frac{+Q}{L-x} \xrightarrow{\quad} \frac{-Q}{x}$ From Eqs. (3-136) and (3-137), $W_e = -$

$$x = x - x_0, \quad \text{Force on } +Q, \quad \bar{F}_{e1} = -\bar{a}_x \frac{\partial W_e}{\partial x} = \bar{a}_x \frac{Q^2}{4\pi \epsilon_0 x^2} = -\bar{F}$$

P. 3-38 a) $V = V_0 = \text{constant}$. $C = \frac{\epsilon x + \epsilon_0(L-x)}{d} w$,

$W_e = \frac{1}{2} CV^2$, $\bar{F}_v = \bar{\nabla} W_e = \bar{a}_x \frac{V_0^2 w}{2d} (\epsilon - \epsilon_0)$.

b) $Q = \text{constant} = CV_0$.

$W_e = \frac{Q^2}{2C}$, $\bar{F}_a = -\bar{\nabla} W_e = \frac{Q^2 d}{2} \frac{\bar{a}_x (\epsilon - \epsilon_0) w}{[\epsilon x + \epsilon_0(L-x)]^2}$
 $= \bar{a}_x \frac{V_0^2 w}{2d} (\epsilon - \epsilon_0)$.

Chapter 4

P. 4-1 $\nabla^2 V = 0 \rightarrow V_d = c_1 y + c_2$, $\bar{E}_d = -\bar{a}_y c_1$, $\bar{D}_d = -\bar{a}_y \epsilon_0 c_1$,

$V_a = c_3 y + c_4$, $\bar{E}_a = -\bar{a}_y c_3$, $\bar{D}_a = -\bar{a}_y \epsilon_0 c_3$.

B.C.: $V_d = 0$ at $y=0$; $V_a = V_0$ at $y=d$;

$V_d = V_a$ at $y=0$; $\bar{D}_d = \bar{D}_a$ at $y=0$.

Solving: $c_1 = \frac{V_0}{(0.8 + 0.2\epsilon_0)d}$, $c_2 = 0$, $c_3 = \frac{\epsilon_0 V_0}{(0.8 + 0.2\epsilon_0)d}$, $c_4 = \frac{(1-\epsilon_0)V_0}{1+0.2\epsilon_0}$.

a) $V_d = \frac{5yV_0}{(4+\epsilon_0)d}$, $\bar{E}_d = -\bar{a}_y \frac{5V_0}{(4+\epsilon_0)d}$.

b) $V_a = \frac{5\epsilon_0 y - 4(\epsilon_0 - 1)d}{(4+\epsilon_0)d} V_0$, $\bar{E}_a = -\bar{a}_y \frac{5\epsilon_0 V_0}{(4+\epsilon_0)d}$.

c) $(\rho_s)_{y=d} = -(D_a)_{y=d} = \frac{5\epsilon_0 \epsilon_0 V_0}{(4+\epsilon_0)d}$.

$(\rho_s)_{y=0} = (D_d)_{y=0} = -\frac{5\epsilon_0 \epsilon_0 V_0}{(4+\epsilon_0)d}$.

P. 4-3 At a point where V is a maximum (minimum) the second partial derivatives of V with respect to x , y and z would all be negative (positive); their sum could not vanish, as required by Laplace's equation.

P. 4-6 $\nabla^2 V = -\frac{A}{\epsilon r} \rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = -\frac{A}{\epsilon r}$,

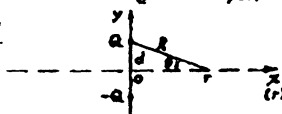
$V = -\frac{A}{\epsilon} r + c_1 \ln r + c_2$

$\begin{cases} V_0 = -\frac{A}{\epsilon} a + c_1 \ln a + c_2 \\ 0 = -\frac{A}{\epsilon} b + c_1 \ln b + c_2 \end{cases}$

$c_1 = \frac{A(b-a) - V_0}{\ln(b/a)}$

$c_2 = \frac{V_0 \ln b + \frac{A}{\epsilon} (a \ln b - b \ln a)}{\ln(b/a)}$

P. 4-7



$\bar{E}|_{y=0} = -\bar{a}_y \frac{Q}{4\pi\epsilon R^2} 2\sin\theta = -\bar{a}_y \frac{Qd}{2\pi\epsilon(d^2+r^2)^{3/2}}$

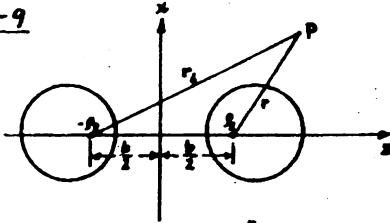
a) $\rho_s = \bar{a}_y \cdot \epsilon \bar{E}|_{y=0} = -\frac{Qd}{2\pi(d^2+r^2)^{3/2}}$

b) $\int_0^{\infty} \rho_s 2\pi r dr = -Q$.

- P.4-8 a) Original point charge Q at $y = d/3$ and images
 Q at $y = (1+6n)d/3$, $n = \pm 1, \pm 2, \dots$
 $-Q$ at $y = (5+6n)d/3$, $n = 0, \pm 1, \pm 2, \dots$

- b) Original line charge ρ_L at $\theta = 30^\circ$ and image line charges ρ_L at $\theta = 150^\circ$ and -90° ; $-\rho_L$ at $\theta = 90^\circ, -30^\circ$ and -150°

P.4-9



From Eq. (4-40), $V = \frac{\rho_L}{2\pi\epsilon_0} \ln \frac{r_i}{r}$

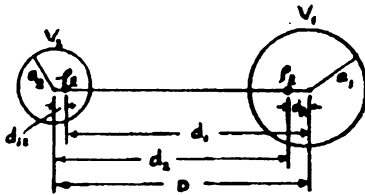
$$r = [x^2 + (z - \frac{b}{2})^2]^{1/2}$$

$$r_i = [x^2 + (z + \frac{b}{2})^2]^{1/2}$$

For equipotential surfaces,

$$\frac{r_i}{r} = \sqrt{K} \rightarrow x^2 + [z - \frac{(K+1)b}{K-1}]^2 = \frac{b^2 K}{(K-1)^2}$$

P.4-10



$$V_1 = -\frac{\rho_L}{2\pi\epsilon_0} \ln \frac{a_1}{d_1}, \quad V_2 = +\frac{\rho_L}{2\pi\epsilon_0} \ln \frac{a_2}{d_2}$$

Capacitance per unit length

$$C = \frac{\rho_L}{V_1 - V_2} = \frac{2\pi\epsilon_0}{\ln \frac{d_1 d_2}{a_1 a_2}}$$

Four equations:

$$a_1^2 = d_1 d_1, \quad a_2^2 = d_2 d_2$$

$$d_1 + d_2 = D, \quad d_2 + d_{i2} = D$$

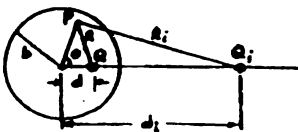
We obtain

$$\frac{d_1 d_2}{a_1 a_2} = \frac{a_1 a_2}{d_{i1} d_{i2}} \quad \text{and} \quad a_1^2 + a_2^2 + d_1 d_1 + d_2 d_2 = D^2$$

$$\frac{d_1 d_2}{a_1 a_2} = \frac{D^2}{2a_1 a_2} - \frac{a_1}{2a_2} - \frac{a_2}{2a_1} + \sqrt{\left(\frac{D^2}{2a_1 a_2} - \frac{a_1}{2a_2} - \frac{a_2}{2a_1}\right)^2 - 1}$$

$$\therefore C = \frac{2\pi\epsilon_0}{\ln \left[\frac{1}{2} \left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1} \right) - \sqrt{\frac{1}{4} \left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1} \right)^2 - 1} \right]} = \frac{2\pi\epsilon_0}{\cosh^{-1} \left[\frac{1}{2} \left(\frac{D^2}{a_1 a_2} - \frac{a_1}{a_2} - \frac{a_2}{a_1} \right) \right]}$$

4-12



$$Q_i = -\frac{b}{d} Q, \quad d_i = \frac{b^2}{d}$$

$$a) V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{b}{dR_i} \right)$$

$$b) \rho_s = -\epsilon_0 \frac{\partial V}{\partial R} \Big|_{R=b} = -\frac{Q(b^3 - d)}{4\pi b(b^2 + d^2)}$$

4-13 At the plane boundary $x=0$: $V_1 = V_2$ and $\epsilon_1 \frac{\partial V_1}{\partial x} = -\epsilon_2 \frac{\partial V_2}{\partial x}$.

Two equations: $\frac{Q_1 - Q_2}{\epsilon_1} = \frac{Q_2 + Q}{\epsilon_2}$ and $Q_1 + Q = Q_2 + Q$.

$$\longrightarrow Q_1 = Q_2 = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1} Q.$$

4-14 The solution $V(x,y) = V_0$ satisfies $\nabla^2 V = 0$ and all boundary conditions. \longrightarrow Unique solution.

4-16 $V(x,y) = \sum_n \sin \frac{n\pi}{a} x [A_n \sinh \frac{n\pi}{a} y + B_n \cosh \frac{n\pi}{a} y]$.

At $y=0$, $V(x,0) = V_2 = \sum_n B_n \sin \frac{n\pi}{a} x \longrightarrow B_n = \begin{cases} \frac{4V_2}{n\pi}, n \text{ odd} \\ 0, n \text{ even.} \end{cases}$

At $y=b$, $V(x,b) = V_1 = \sum_n \sin \frac{n\pi}{a} x [A_n \sinh \frac{n\pi b}{a} + B_n \cosh \frac{n\pi b}{a}]$

$$\longrightarrow A_n \sinh \frac{n\pi b}{a} + B_n \cosh \frac{n\pi b}{a} = \begin{cases} \frac{4V_1}{n\pi}, n \text{ odd} \\ 0, n \text{ even.} \end{cases}$$

$$\therefore A_n = \begin{cases} \frac{4}{n\pi \sinh \frac{n\pi b}{a}} (V_1 - V_2 \cosh \frac{n\pi b}{a}), n \text{ odd} \\ 0, n \text{ even.} \end{cases}$$

4-13 The solution is the superposition of that for Example 4-9 and that for Fig. 4-12 rotated 90° in the clockwise direction. (In both cases V_0 should be replaced by $V_0/2$.)

Inside: $V(r,\phi) = \frac{2V_0}{\pi} \sum_{n \text{ odd}} \frac{1}{n} \left(\frac{r}{b}\right)^n \left[\sin n\phi + \sin n\left(\phi + \frac{\pi}{2}\right) \right], r < b$

Outside: $V(r,\phi) = \frac{2V_0}{\pi} \sum_{n \text{ odd}} \frac{1}{n} \left(\frac{b}{r}\right)^n \left[\sin n\phi + \sin n\left(\phi + \frac{\pi}{2}\right) \right], r > b.$

4-19 $V(r,\phi) = -E_0 r \cos \phi + \sum_{n=1}^{\infty} B_n r^{-n} \cos n\phi$. (At $r \gg b$, $\bar{E} = \bar{a}_z E_0, V = -E_0 r \cos \phi$)

At $r=b$, $V(b,\phi) = -E_0 b \cos \phi + \sum_{n=1}^{\infty} B_n b^{-n} \cos n\phi$

$$\longrightarrow B_1 = E_0 b^2; B_n = 0 \text{ for } n \neq 1.$$

Outside the cylinder: $V(r,\phi) = -E_0 r \left(1 - \frac{b^2}{r^2}\right) \cos \phi,$

$$\bar{E}(r,\phi) = -\nabla V = \bar{a}_z E_0 \left(\frac{b^2}{r^2} + 1\right) \cos \phi + \bar{a}_\phi E_0 \left(\frac{b^2}{r^2}\right) \sin \phi.$$

4-20 For $r \geq b$, $V_o(r,\phi) = -E_0 r \cos \phi + \sum_{n=1}^{\infty} B_n r^{-n} \cos n\phi,$

$r \leq b$, $V_i(r,\phi) = \sum_{n=1}^{\infty} A_n r^n \cos n\phi.$

At $r=b$: $V_o(b,\phi) = V_i(b,\phi) \longrightarrow -E_0 b + B_1 b^{-1} = A_1 b; B_1 b^{-2} = A_1 b^2, n=1.$

$$-\frac{\partial V_o}{\partial r} \Big|_{r=b} = -E_0 + \frac{\partial V_i}{\partial r} \Big|_{r=b} \longrightarrow E_0 + B_1 b^{-2} = -\epsilon_r A_1; n B_1 b^{-(n+1)} = -\epsilon_r n A_1 b^{n-1}, n \neq 1.$$

Solving: $A_1 = -\frac{2E_0}{\epsilon_r + 1}$, $B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 1} b^2 E_0$,
 $A_n = B_n = 0$ for $n \neq 1$.

$$V_0(r, \phi) = -\left(1 - \frac{\epsilon_r - 1}{\epsilon_r + 1} \frac{b^2}{r^2}\right) E_0 r \cos \phi.$$

$$V_i(r, \phi) = -\frac{2}{\epsilon_r + 1} E_0 r \cos \phi.$$

$$\vec{E} = -\vec{\nabla} V = -\vec{a}_r \frac{\partial V}{\partial r} - \vec{a}_\phi \frac{\partial V}{r \partial \phi}.$$

$$\vec{E}_0 = \vec{a}_x E_0 = \frac{\epsilon_r - 1}{\epsilon_r + 1} \left(\frac{b}{r}\right)^2 E_0 (\vec{a}_r \cos \phi + \vec{a}_\phi \sin \phi).$$

$$\vec{E}_i = \frac{2}{\epsilon_r + 1} \vec{a}_x E_0 = \frac{2}{\epsilon_r + 1} (\vec{a}_r \cos \phi - \vec{a}_\phi \sin \phi).$$

P. 4-21 Starting from Eq. (4-134) and applying the boundary condition $V(b, \theta) = V_0$:

$$V_0 = \frac{B_0}{b} + \left(\frac{B_1}{b^2} - E_0\right) \cos \theta - \sum_{n=2}^{\infty} B_n b^{-(n+1)} P_n(\cos \theta), \quad R \geq b.$$

$$\longrightarrow B_0 = b V_0; \quad B_1 = E_0 b^2; \quad B_n = 0 \text{ for } n \geq 2.$$

$$\therefore V(R, \theta) = \frac{b}{R} V_0 - E_0 \left[1 - \left(\frac{b}{R}\right)^2\right] R \cos \theta, \quad R \geq b.$$

$$\vec{E}(R, \theta) = \vec{a}_R \left\{ \frac{b V_0}{R^2} + E_0 \left[1 + 2\left(\frac{b}{R}\right)^2\right] \cos \theta \right\} \\ - \vec{a}_\theta E_0 \left[1 - \left(\frac{b}{R}\right)^2\right] R \sin \theta, \quad R \geq b.$$

$$P_s(\theta) = \epsilon_0 E_n \Big|_{R=b} = \epsilon_0 \frac{V_0}{b} + 3 \epsilon_0 E_0 \cos \theta.$$

P. 4-22 $V_i(R, \theta) = \sum_{n=0}^{\infty} A_n R^n P_n(\cos \theta), \quad R \leq b.$

$$V_o(R, \theta) = \sum_{n=0}^{\infty} (B_n R^n + C_n R^{-(n+1)}) P_n(\cos \theta), \quad R \geq b.$$

For $R \gg b$, $V_o(R, \theta) = -E_0 z = -E_0 R \cos \theta \rightarrow B_1 = -E_0; B_n = C_n = 0$

$$\therefore V_o(R, \theta) = -E_0 R \cos \theta + C_1 R^{-2} \cos \theta. \quad \text{for } n \neq 1.$$

$$\text{B.C.: } V_i(b, \theta) = V_o(b, \theta) \rightarrow A_1 b = -E_0 b + C_1 b^{-2} \quad \left. \vphantom{V_i(b, \theta) = V_o(b, \theta)} \right\} A_1 = -\frac{3E_0}{\epsilon_r + 2}$$

$$\epsilon_r \frac{\partial V_i}{\partial R} \Big|_{R=b} = \frac{\partial V_o}{\partial R} \Big|_{R=b} \rightarrow \epsilon_r A_1 = -E_0 - 2C_1 b^{-3} \quad \left. \vphantom{\epsilon_r \frac{\partial V_i}{\partial R} \Big|_{R=b}} \right\} C_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0 b^3$$

$$\therefore V_i(R, \theta) = -\frac{3E_0}{\epsilon_r + 2} R \cos \theta, \quad V_o(R, \theta) = -E_0 R \cos \theta - \frac{(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^2} E_0 \cos \theta.$$

$$\vec{E}_i(R, \theta) = -\vec{\nabla} V_i = \frac{3E_0}{\epsilon_r + 2} (\vec{a}_x \cos \theta - \vec{a}_\theta \sin \theta) = \vec{a}_x \frac{3E_0}{\epsilon_r + 2}.$$

$$\vec{E}_o(R, \theta) = -\vec{\nabla} V_o = \vec{a}_R \left[1 + \frac{2(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^3}\right] E_0 \cos \theta - \vec{a}_\theta \left[1 - \frac{(\epsilon_r - 1)b^3}{(\epsilon_r + 2)R^3}\right] E_0 \sin \theta.$$

Chapter 5

$$R_1 = \text{Resistance per unit length of core} = \frac{l}{\sigma S_1} = \frac{l}{\sigma \pi a^2}$$

$$R_2 = \text{Resistance per unit length of coating} = \frac{l}{0.1 \sigma S_2}$$

Let $b =$ Thickness of coating.

$$S_2 = \pi(a+b)^2 - \pi a^2 = \pi(2ab + b^2).$$

a) $R_1 = R_2 \rightarrow b = (\sqrt{11} - 1)a = 2.32a.$

b) $I_1 = I_2 = I/2 : J_1 = \frac{I}{2\pi a^2}, E_1 = \frac{J_1}{\sigma} = \frac{I}{2\pi a^2 \sigma}$

$$J_2 = \frac{I}{2S_2} = \frac{I}{20\pi a^2}, E_2 = \frac{J_2}{0.1\sigma} = \frac{I}{2\pi a^2 \sigma} = E_1.$$

1) $I_1 = 0.7 \text{ (A)}, P_{R1} = 0.163 \text{ (W)}; I_2 = 0.140 \text{ (A)}, P_{R2} = 0.392 \text{ (W)};$

$I_3 = 0.093 \text{ (A)}, P_{R3} = 0.261 \text{ (W)}; I_4 = 0.233 \text{ (A)}, P_{R4} = 0.436 \text{ (W)};$

$I_5 = 0.467 \text{ (A)}, P_{R5} = 2.178 \text{ (W)}.$

5) $\rho = \rho_0 e^{-(\sigma/\epsilon)t}, \rho_0 = \frac{Q_0}{(4\pi/3)b^3} = \frac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239 \text{ (C/m}^3\text{)}.$

a) $R < b, \bar{E}_i = \bar{a}_R \frac{(4\pi/3)R^3 \rho}{4\pi \epsilon R^2} = \bar{a}_R \frac{\rho_0 R}{3\epsilon} e^{-(\sigma/\epsilon)t}$
 $= \bar{a}_R 7.5 \times 10^9 R e^{-9.42 \times 10^{11} t} \text{ (V/m)};$

$R > b, \bar{E}_o = \bar{a}_R \frac{Q_0}{4\pi \epsilon R^2} = \bar{a}_R \frac{9}{R^2} \times 10^6 \text{ (V/m)}.$

b) $R < b, \bar{J}_i = \sigma \bar{E}_i = \bar{a}_R 7.5 \times 10^{10} R e^{-9.42 \times 10^{11} t} \text{ (A/m}^2\text{)};$

$R > b, \bar{J}_o = 0.$

6) a) $e^{-(\sigma/\epsilon)t} = \frac{\rho}{\rho_0} = 0.01 \rightarrow t = \frac{\ln 100}{(\sigma/\epsilon)} = 4.88 \times 10^{-12} \text{ (s)}$
 $= 4.88 \text{ (ps)}.$

b) $W_i = \frac{\epsilon}{2} \int_V E_i^2 dv' = \frac{2\pi \rho_0 b^2}{45 \epsilon} e^{-2(\sigma/\epsilon)t} = (W_i)_0 [e^{-(\sigma/\epsilon)t}]^2$

$\therefore \frac{W_i}{(W_i)_0} = [e^{-(\sigma/\epsilon)t}]^2 = 0.01^2 = 10^{-4}.$ Energy dissipated as heat loss.

c) Electrostatic energy stored outside the sphere

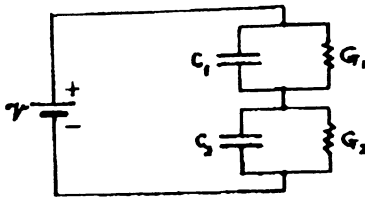
$$W_o = \frac{\epsilon}{2} \int_b^\infty E_o^2 4\pi R^2 dR = \frac{Q_0^2}{8\pi \epsilon_0 b} = 45 \text{ (kJ)}. \text{ Constant.}$$

7) a) $R = \frac{l}{\sigma S} = \frac{l}{I} \rightarrow \sigma = \frac{I}{S R} = 3.537 \times 10^7 \text{ (S/m)}.$

b) $E = \frac{V}{l} = 6 \times 10^{-3} \text{ (V/m)}, \text{ or } E = \frac{J}{\sigma} = \frac{I}{\sigma S}.$

c) $P = VI = 1 \text{ (W)}.$

P.5-8



a) $C_1 = \frac{\epsilon_1 S}{d_1}, G_1 = \frac{\sigma_1 S}{d_1}$

$C_2 = \frac{\epsilon_2 S}{d_2}, G_2 = \frac{\sigma_2 S}{d_2}$

b) $P = V^2 G = V^2 \frac{G_1 G_2}{G_1 + G_2}$

$= V^2 S \frac{\sigma_1 \sigma_2}{\sigma_1 d_2 + \sigma_2 d_1}$

P.5-9 a) $G_1 = \frac{2\pi\sigma_1}{\ln(c/a)}, G_2 = \frac{2\pi\sigma_2}{\ln(b/c)}$

$I = V G = V \frac{G_1 G_2}{G_1 + G_2} = \frac{2\pi\sigma_1 \sigma_2 V}{\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)}$

$J_1 = J_2 = \frac{I}{2\pi r L} = \frac{\sigma_1 \sigma_2 V}{r L [\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}$

b) $\rho_{sa} = \epsilon_1 E_1 \Big|_{r=a} = \frac{\epsilon_1 \sigma_2 V}{a L [\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}$

$\rho_{sb} = -\epsilon_2 E_2 \Big|_{r=b} = -\frac{\epsilon_2 \sigma_1 V}{b L [\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}$

$\rho_{sc} = -(\epsilon_1 E_1 - \epsilon_2 E_2) \Big|_{r=c} = \frac{(\epsilon_1 \sigma_1 - \epsilon_2 \sigma_2) V}{c L [\sigma_1 \ln(b/c) + \sigma_2 \ln(c/a)]}$

P.5-10 $\nabla^2 V = 0 \rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 0$

$V(r) = c_1 \ln r + c_2; B.C.: V(a) = V_0, V(b) = 0$

$V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}$

$\vec{E}(r) = -\vec{a}_r \frac{\partial V}{\partial r} = \vec{a}_r \frac{V_0}{r \ln(b/a)}, \vec{J} = \sigma \vec{E}$

$I = \int_S \vec{J} \cdot d\vec{s} = \int_0^{2\pi} \int_0^h \vec{J} \cdot (\vec{a}_r h r d\phi) = \frac{\pi \sigma h V_0}{2 \ln(b/a)}$

$R = \frac{V_0}{I} = \frac{2 \ln(b/a)}{\pi \sigma h}$

P.5-11 Assume a current I between the spherical surfaces.

$\vec{J} = \vec{a}_r \frac{I}{4\pi R^2} = \sigma \vec{E}$

$V_0 = -\int_{R_1}^{R_2} \vec{E} \cdot d\vec{R} = \int_{R_1}^{R_2} \frac{I dR}{4\pi R^2 \sigma} = \frac{I}{4\pi \sigma_0} \int_{R_1}^{R_2} \frac{dR}{R^2 (1+k/R)}$

$= \frac{I}{4\pi \sigma_0} \int_{R_1}^{R_2} \frac{1}{k} \left(\frac{1}{R} - \frac{1}{R+k} \right) dR = \frac{I}{4\pi \sigma_0 k} \ln \frac{R}{R+k}$

$R = \frac{V_0}{I} = \frac{1}{4\pi \sigma_0 k} \ln \frac{R_2 (R_1 + k)}{R_1 (R_2 + k)}$

P. 5-12 Assume I. $\bar{J}(R) = \bar{a}_R \frac{I}{S(R)}$

$$S(R) = \int_0^{2\pi} \int_0^{\theta_0} R^2 \sin\theta d\theta d\phi = 2\pi R^2 (1 - \cos\theta_0)$$

$$\bar{E}(R) = \frac{\bar{J}(R)}{\sigma} = \bar{a}_R \frac{I}{2\pi\sigma R^2 (1 - \cos\theta_0)}$$

$$V_0 = - \int_{R_1}^{R_2} \bar{E}(R) dR = \frac{I(R_2 - R_1)}{2\pi\sigma R_1 R_2 (1 - \cos\theta_0)}$$

$$\therefore R = \frac{V_0}{I} = \frac{R_2 - R_1}{2\pi\sigma R_1 R_2 (1 - \cos\theta_0)}$$

P. 5-13 $\bar{\nabla} \cdot \bar{J} = \bar{\nabla} \cdot (\sigma \bar{E}) = \sigma \bar{\nabla} \cdot \bar{E} + (\bar{\nabla} \sigma) \cdot \bar{E} = 0$

$$\bar{E} = \bar{a}_R E, \quad \bar{\nabla} \cdot \bar{E} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 E); \quad \bar{\nabla} \sigma = \bar{a}_R \frac{\partial \sigma}{\partial R} = -\bar{a}_R \frac{\sigma}{R^2}$$

$$\rightarrow R \frac{\partial E}{\partial R} = -E, \quad \bar{E} = \bar{a}_R \frac{c}{R}$$

$$V_0 = - \int_{R_1}^{R_2} \bar{E} \cdot d\bar{R} = - \int_{R_1}^{R_2} \frac{c}{R} dR = c \ln \frac{R_1}{R_2}$$

$$c = \frac{V_0}{\ln(R_1/R_2)}, \quad \bar{E} = \bar{a}_R \frac{V_0}{R \ln(R_1/R_2)}$$

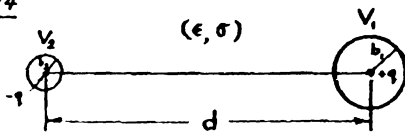
$$I = \int_S \bar{J} \cdot d\bar{s} = \int_S \sigma \bar{E} \cdot d\bar{s}$$

$$= \int_0^{2\pi} \int_0^{\theta_0} \left(\frac{\sigma R_1}{R} \right) \left[\frac{V_0}{R \ln(R_1/R_2)} \right] R^2 \sin\theta d\theta d\phi$$

$$= \frac{2\pi\sigma R_1 V_0 (1 - \cos\theta_0)}{\ln(R_1/R_2)}$$

$$\therefore R = \frac{V_0}{I} = \frac{\ln(R_1/R_2)}{2\pi\sigma R_1 (1 - \cos\theta_0)}$$

P. 5-14



Assume charges $+q$ and $-q$ to concentrate at the centers of spheres 1 and 2 respectively.

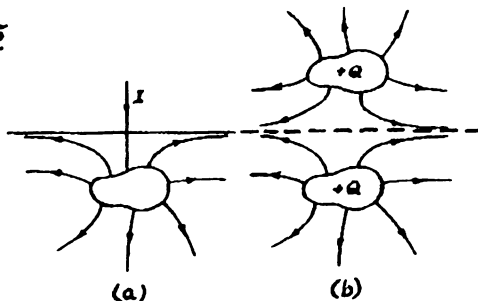
$$d \gg b_1, d \gg b_2. \quad V_1 \approx \frac{q}{4\pi\epsilon} \left(\frac{1}{b_1} - \frac{1}{d-b_1} \right),$$

$$V_2 \approx \frac{q}{4\pi\epsilon} \left(\frac{1}{d-b_2} - \frac{1}{b_2} \right).$$

$$C = \frac{q}{V_1 - V_2} \approx \frac{4\pi\epsilon}{\frac{1}{b_1} + \frac{1}{b_2} - \frac{1}{d-b_1} - \frac{1}{d-b_2}} = G \frac{\epsilon}{\sigma} = \frac{\epsilon}{R\sigma},$$

$$R = \frac{1}{4\pi\sigma} \left(\frac{1}{b_1} + \frac{1}{b_2} - \frac{1}{d-b_1} - \frac{1}{d-b_2} \right) = \frac{1}{4\pi\sigma} \left(\frac{1}{b_1} + \frac{1}{b_2} - \frac{2}{d} \right).$$

P.5-15



The current flow pattern of the lower half of Fig. (b) if both the conductor and its image are fed with the same current is exactly the same as that of Fig. (a). All boundary conditions are satisfied.

The streamlines are similar to the electric field lines of a conductor and its image, both carrying a charge $+Q$, in the electrostatic case.

P.5-16 According to P.5-15, the current flow pattern would be the same as that of a whole sphere in unbounded earth medium. Hence the current lines are radial. Assume a current I .

$$\vec{J} = \bar{a}_r \frac{I}{2\pi R^2}, \quad \vec{E} = \bar{a}_r \frac{I}{2\pi\sigma R^2}$$

$$V_0 = -\int_{\infty}^b E \, dR = -\frac{I}{2\pi\sigma} \int_{\infty}^b \frac{dR}{R^2} = \frac{I}{2\pi\sigma b}$$

$$R = \frac{V_0}{I} = \frac{1}{2\pi\sigma b} = \frac{1}{2\pi (10^{-6})(25 \times 10^{-3})} = 6.36 \times 10^6 (\Omega)$$

P.5-17 The boundary conditions at $y=0$ and $y=b$ require that $Y(y) \sim \cos(\frac{n\pi}{b}y)$; the boundary condition at $x=0$ indicates that $X(x) \sim \sinh(\frac{n\pi}{b}x)$. Thus,

$$a) \quad V(x,y) = \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}x\right) \cos\left(\frac{n\pi}{b}y\right).$$

$$\begin{aligned} \text{B.C. at } x=a: V(a,y) = V_0 &= \sum_{n=0}^{\infty} C_n \sinh\left(\frac{n\pi}{b}a\right) \cos\left(\frac{n\pi}{b}y\right) \\ &= \sum_{n=0}^{\infty} B_n \cos\left(\frac{n\pi}{b}y\right). \end{aligned}$$

$$\int_0^b [\quad] \cos\left(\frac{n\pi}{b}y\right) dy: \quad 0 = B_n \left(\frac{b}{2}\right) \rightarrow B_n = 0 \text{ for } n \neq 0$$

$$\text{For } n=0, \quad V_0 = B_0 \rightarrow C_0 = \frac{V_0}{\sinh(n\pi a/b)}$$

$$V(x,y) = V_0 \left[\frac{\sinh(n\pi x/b)}{\sinh(n\pi a/b)} \cos\left(\frac{n\pi}{b}y\right) \right]_{n=0} = \frac{V_0}{a} x.$$

$$b) \bar{J} = -\sigma \bar{E} = -\sigma \bar{\nabla} V = -\bar{a}_x \frac{\sigma V_0}{a}$$

P.5-18 $V(r, \phi) = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-n}) (C_n \cos n\phi + D_n \sin n\phi)$.

B.C.: $V(r, \phi) = V(r, -\phi) \longrightarrow D_n = 0$.

$r \rightarrow \infty, V(r, \phi) = -\frac{1}{\sigma} J_0 r \cos \phi \longrightarrow A_n = C_n = 0$ for $n \neq 1$.

Write $V(r, \phi) = (K_1 r + \frac{K_2}{r}) \cos \phi$. $K_1 = A_1 C_1, K_2 = B_1 C_1$.

$$K_1 = -\frac{J_0}{\sigma}$$

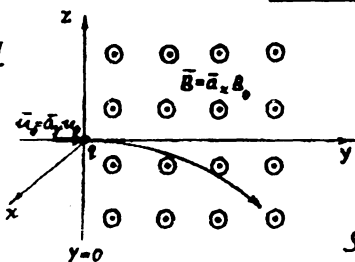
B.C.: $\frac{\partial V}{\partial r} \Big|_{r=b} = 0 \longrightarrow K_1 - \frac{K_2}{b^2} = 0, K_2 = b^2 K_1 = -\frac{J_0 b^2}{\sigma}$.

$$\therefore V(r, \phi) = -\frac{J_0}{\sigma} (r + \frac{b^2}{r}) \cos \phi$$

$$\begin{aligned} \bar{J} &= -\sigma \bar{\nabla} V = -\sigma (\bar{a}_r \frac{\partial V}{\partial r} + \bar{a}_\phi \frac{\partial V}{r \partial \phi}) \\ &= \bar{a}_r J_0 (1 - \frac{b^2}{r^2}) \cos \phi - \bar{a}_\phi J_0 (1 + \frac{b^2}{r^2}) \sin \phi \\ &= J_0 (\bar{a}_r \cos \phi - \bar{a}_\phi \sin \phi) - \frac{J_0 b^2}{r^2} (\bar{a}_r \cos \phi + \bar{a}_\phi \sin \phi) \\ &= \bar{a}_x J_0 - \frac{J_0 b^2}{r^2} (\bar{a}_r \cos \phi + \bar{a}_\phi \sin \phi), \quad r > a. \\ \bar{J} &= 0, \quad r < b. \end{aligned}$$

Chapter 6

P.6-1



$$\frac{du_y}{dt} = \frac{q B_0}{m} u_z = \omega_0 u_z \quad (1)$$

$$\frac{du_x}{dt} = -\frac{q B_0}{m} u_y = -\omega_0 u_y \quad (2) \quad \omega_0 = \frac{q B_0}{m}$$

Combining (1) and (2):

$$\frac{d^2 u_x}{dt^2} + \omega_0^2 u_x = 0$$

Solution: $u_x = A \cos \omega_0 t + B \sin \omega_0 t$.

At $t=0, u_x = 0 \longrightarrow A = 0,$

$$u_x = B \sin \omega_0 t$$

Substituting u_x in (2): $u_y = -B \cos \omega_0 t$.

At $t=0, u_y = u_0 \longrightarrow B = -u_0$.

$$\therefore u_y = u_0 \cos \omega_0 t \longrightarrow y = \frac{u_0}{\omega_0} \sin \omega_0 t \quad (t=0, y=0)$$

$$\begin{aligned} u_z &= -u_0 \sin \omega_0 t \longrightarrow z = \frac{u_0}{\omega_0} \cos \omega_0 t + C \quad (t=0, z=0 \longrightarrow C = -\frac{u_0}{\omega_0}) \\ &= -\frac{u_0}{\omega_0} (1 - \cos \omega_0 t) \end{aligned}$$

We obtain $y^2 + (z + \frac{u_0}{\omega_0})^2 = (\frac{u_0}{\omega_0})^2$ --- Eq. of a shifted circle.

P.6-2 $\frac{\partial \bar{u}}{\partial t} = -\frac{|e|\hbar}{m} (\bar{E} + \bar{u} \times \bar{B})$

a) $\bar{E} = \bar{a}_z E_0, \bar{B} = \bar{a}_x B_0.$

$$\left. \begin{aligned} \frac{\partial u_x}{\partial t} &= 0 \\ \frac{\partial u_y}{\partial t} &= -\frac{|e|\hbar}{m} B_0 u_x \\ \frac{\partial u_z}{\partial t} &= -\frac{|e|\hbar}{m} (E_0 - B_0 u_y) \end{aligned} \right\} \begin{cases} u_x = 0 \\ u_y = (u_0 - \frac{E_0}{B_0}) \cos \omega_0 t + \frac{E_0}{B_0} \\ u_z = (\frac{E_0}{B_0} - u_0) \sin \omega_0 t, \end{cases}$$

$$\omega_0 = \frac{|e|\hbar}{m} B_0$$

If the electron is injected at the origin ($x=y=z=0$) at $t=0$,

we have $x=0, y = \frac{c}{\omega_0} \sin \omega_0 t + \frac{E_0}{B_0} t, z = -\frac{c}{\omega_0} (1 - \cos \omega_0 t).$

Equation of motion:

$$(y - \frac{E_0}{B_0} t)^2 + (z + \frac{c}{\omega_0})^2 = (\frac{c}{\omega_0})^2.$$

$$c = u_0 - \frac{E_0}{B_0}$$

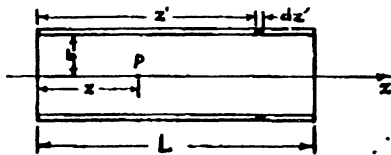
If $E_0/B_0 = u_0, u_x = u_z = 0, u_y = u_0; x = z = 0, y = u_0 t.$

b) $\bar{E} = -\bar{a}_z E_0, \bar{B} = -\bar{a}_x B_0.$

$$\left. \begin{aligned} \frac{\partial u_x}{\partial t} &= \frac{|e|\hbar}{m} B_0 u_y = \omega_0 u_y \\ \frac{\partial u_y}{\partial t} &= -\omega_0 u_x \\ \frac{\partial u_z}{\partial t} &= \frac{|e|\hbar}{m} E_0. \end{aligned} \right\} \begin{cases} \text{Circular motion} \\ \text{(See P.6-1)} \end{cases} \left. \begin{aligned} & \\ & \end{aligned} \right\} \begin{cases} \text{Helical motion with} \\ \text{constant acceleration} \\ |e|\hbar E_0/m \text{ in } z \text{ direction.} \end{cases}$$

P.6-3 Straightforward application of Ampère's circuital law.

P.6-4



From Example 6-6, Eq. (6-38)

$$dB = \frac{\mu_0 I b^2}{2 [(z'-z)^2 + b^2]^{3/2}} \left(\frac{N}{L} dz' \right)$$

$$\therefore B = \frac{\mu_0 N I b^2}{2L} \int_0^L \frac{dz'}{[(z'-z)^2 + b^2]^{3/2}}$$

or, $B = \frac{\mu_0 N I}{2L} \left[\frac{L-z}{\sqrt{(L-z)^2 + b^2}} + \frac{z}{\sqrt{z^2 + b^2}} \right] \rightarrow \mu_0 \left(\frac{N}{L} \right) I$ as $L \rightarrow \infty.$

Direction of B is determined by the right-hand rule.

P.6-5 Eq. (6-22): $\bar{A} = \frac{\mu_0}{4\pi} \int_V \frac{\bar{J}}{R} dv'$

$$\bar{B} = \bar{\nabla} \times \bar{A} = \frac{\mu_0}{4\pi} \int_V \bar{\nabla} \times \left(\frac{1}{R} \bar{J} \right) dv' = \frac{\mu_0}{4\pi} \int_V \left[\frac{1}{R} \bar{\nabla} \times \bar{J} + \left(\bar{\nabla} \frac{1}{R} \right) \times \bar{J} \right] dv'$$

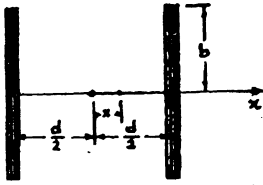
$\bar{\nabla} \times \bar{J} = 0$ because the curl operation $\bar{\nabla} \times$ respect to unprimed coordinates at the and \bar{J} is a function of primed (source

$$\vec{\nabla} \frac{1}{R} = -\vec{a}_R \frac{1}{R^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{-\vec{a}_R \times \vec{J}}{R^2} dv' = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J} \times \vec{a}_R}{R^2} dv'$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{B} &= \frac{\mu_0}{4\pi} \int_V \vec{\nabla} \cdot \left[\vec{\nabla} \left(\frac{1}{R} \right) \times \vec{J} \right] dv' \\ &= \frac{\mu_0}{4\pi} \int_V \left[\vec{J} \cdot \underbrace{(\vec{\nabla} \times \vec{\nabla} \frac{1}{R})}_0 - \underbrace{(\vec{\nabla} \frac{1}{R}) \cdot \vec{\nabla} \times \vec{J}}_0 \right] dv', \quad \text{from P. 2-23.} \\ &= 0. \end{aligned}$$

P. 6-6



Use Eq. (6-38).

$$B_x = \frac{N\mu_0 I b}{2} \left\{ \frac{1}{\left[\left(\frac{d}{2} + x \right)^2 + b^2 \right]^{3/2}} + \frac{1}{\left[\left(\frac{d}{2} - x \right)^2 + b^2 \right]^{3/2}} \right\}$$

a) At $x=0$, $B_x = \frac{N\mu_0 I b}{\left[(d/2)^2 + b^2 \right]^{3/2}}$

b) $\frac{dB_x}{dx} = \frac{N\mu_0 I b}{2} \left\{ -\frac{3}{2} \frac{2\left(\frac{d}{2} + x\right)}{\left[\left(\frac{d}{2} + x \right)^2 + b^2 \right]^{5/2}} + \frac{3}{2} \frac{2\left(\frac{d}{2} - x\right)}{\left[\left(\frac{d}{2} - x \right)^2 + b^2 \right]^{5/2}} \right\}$

At the midpoint, $x=0$, $\frac{dB_x}{dx} = 0$.

c) $\frac{d^2 B_x}{dx^2} = -\frac{3N\mu_0 I b}{2} \left\{ \frac{1}{\left[\left(\frac{d}{2} + x \right)^2 + b^2 \right]^{5/2}} - \frac{5\left(\frac{d}{2} + x\right)^2}{\left[\left(\frac{d}{2} + x \right)^2 + b^2 \right]^{7/2}} + \frac{1}{\left[\left(\frac{d}{2} - x \right)^2 + b^2 \right]^{5/2}} - \frac{5\left(\frac{d}{2} - x\right)^2}{\left[\left(\frac{d}{2} - x \right)^2 + b^2 \right]^{7/2}} \right\}$

At $x=0$, $\frac{d^2 B_x}{dx^2} = -\frac{6N\mu_0 I b}{2} \left\{ \frac{(d/2)^2 + b^2 - 5(d/2)^2}{\left[(d/2)^2 + b^2 \right]^{7/2}} \right\}$,

which vanishes if $b^2 - 4(d/2)^2 = 0$, or $b=d$.

P. 6-7



Use Eq. (6-35) for a wire of length $2L$.

$$\vec{B} = \vec{a}_\phi \frac{\mu_0 I L}{2\pi r \sqrt{L^2 + r^2}}$$

In this problem, $\alpha = \frac{2\pi}{2N} = \frac{\pi}{N}$, $\vec{B} = \vec{a}_n N \left(\frac{\mu_0 I L}{2\pi r b} \right) = \vec{a}_n \frac{\mu_0 N I}{2\pi b} \tan \frac{\pi}{N}$.

$\frac{L}{r} = \tan \alpha = \tan \frac{\pi}{N}$,

When N is very large, $\tan \frac{\pi}{N} \approx \frac{\pi}{N}$, $\vec{B} \rightarrow \vec{a}_n \frac{\mu_0 I}{2b}$,

which is the same as Eq. (6-38) with $z=0$.

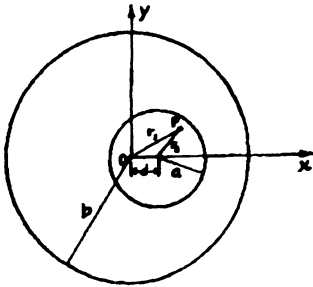
P.6-8 $B_{\phi} = \frac{\mu_0 N I}{2\pi r}$

$$\Phi = \int B_{\phi} ds = \frac{\mu_0 N I}{2\pi} \int_a^b \frac{1}{r} h dr = \frac{\mu_0 N I h}{2\pi} \ln \frac{b}{a}$$

If B_{ϕ} at $r = \frac{a+b}{2}$ is used, $\Phi' = \frac{\mu_0 N I h}{\pi} \left(\frac{b-a}{b+a} \right)$

$$\% \text{ error} = \frac{\Phi' - \Phi}{\Phi} \times 100\% = \left[\frac{2(b-a)}{(b+a) \ln(b/a)} - 1 \right] \times 100\%$$

P.6-9



$$\bar{J} = \bar{a}_z J, \quad \oint \bar{B} \cdot d\bar{l} = \mu_0 I$$

If there were no hole,

$$2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$$

$$B_{\phi 1} = \frac{\mu_0 r_1}{2} J \quad \begin{cases} B_{x1} = -\frac{\mu_0 J}{2} y_1 \\ B_{y1} = +\frac{\mu_0 J}{2} x_1 \end{cases}$$

For $-J$ in the hole portion:

$$B_{\phi 2} = -\frac{\mu_0 r_2}{2} J \quad \begin{cases} B_{x2} = +\frac{\mu_0 J}{2} y_2 \\ B_{y2} = -\frac{\mu_0 J}{2} x_2 \end{cases}$$

Superposing $B_{\phi 1}$ and $B_{\phi 2}$

and noting that $y_1 = y_2$ and $x_1 = x_2 + d$, we have

$$B_x = B_{x1} + B_{x2} = 0 \quad \text{and} \quad B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d$$

P.6-11 $\bar{B} = \nabla \times \bar{A}, \quad \bar{B} = \bar{a}_{\phi} B = \bar{a}_{\phi} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) = -\bar{a}_{\phi} \frac{\partial A_z}{\partial r}$

For $0 \leq r \leq b$, Eq. (6-10) gives $\bar{B}_1 = \bar{a}_{\phi} \frac{\mu_0 I}{2\pi b^2} r$

For $r \geq b$, Eq. (6-11) gives $\bar{B}_2 = \bar{a}_{\phi} \frac{\mu_0 I}{2\pi} \frac{1}{r}$

Integrating, $\bar{A}_1 = \bar{a}_z \left[-\frac{\mu_0 I}{4\pi} \left(\frac{r}{b} \right)^2 + c_1 \right], \quad 0 \leq r \leq b$

$$\bar{A}_2 = \bar{a}_z \left[-\frac{\mu_0 I}{2\pi} \ln r + c_2 \right], \quad r \geq b$$

At $r=b$, $\bar{A}_1 = \bar{A}_2 \rightarrow c_2 = -\frac{\mu_0 I}{4\pi} + \frac{\mu_0 I}{2\pi} \ln b + c_1$,

$$\therefore \bar{A}_2 = \bar{a}_z \left\{ -\frac{\mu_0 I}{4\pi} \left[\ln \left(\frac{r}{b} \right)^2 + 1 \right] + c_1 \right\}, \quad r \geq b$$

P.6-12 Eq. (6-34) for one wire: $\bar{A} = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L}$

For two wires with equal and opposite currents:

a) $\bar{A} = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r_1^2} + L}{\sqrt{L^2 + r_1^2} - L} \frac{\sqrt{L^2 + r_2^2} - L}{\sqrt{L^2 + r_2^2} + L} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln \left[\frac{r_1}{r_2} \frac{\sqrt{L^2 + r_2^2} + L}{\sqrt{L^2 + r_1^2} + L} \right]$

b) For a very long two-wire transmission line, $L \rightarrow \infty$:

$$\bar{A} = \bar{a}_z \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_1}{r_2}\right) = \bar{a}_z \frac{\mu_0 I}{4\pi} \ln\left(\frac{(\frac{d}{2} + y)^2 + x^2}{(\frac{d}{2} - y)^2 + x^2}\right)$$

$$\begin{aligned} \text{c) } \bar{B} &= \bar{\nabla} \times \bar{A} = \bar{a}_x \frac{\partial A_z}{\partial y} - \bar{a}_y \frac{\partial A_z}{\partial x} \\ &= \bar{a}_z \frac{\mu_0 I}{2\pi} \left[\frac{\frac{d}{2} + y}{(\frac{d}{2} + y)^2 + x^2} - \frac{\frac{d}{2} - y}{(\frac{d}{2} - y)^2 + x^2} \right] \\ &\quad - \bar{a}_y \frac{\mu_0 I}{2\pi} \left[\frac{x}{(\frac{d}{2} + y)^2 + x^2} - \frac{x}{(\frac{d}{2} - y)^2 + x^2} \right] = \frac{\mu_0 I}{2\pi} \left[\bar{a}_x \frac{1}{r_1} - \bar{a}_x \frac{1}{r_2} \right] \end{aligned}$$

P.6-14 Apply divergence theorem to $(\bar{F} \times \bar{C})$:

$$\int_V \bar{\nabla} \cdot (\bar{F} \times \bar{C}) dv = \oint_S (\bar{F} \times \bar{C}) \cdot d\bar{s}$$

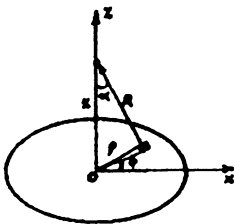
$$\begin{aligned} \text{Now } \bar{\nabla} \cdot (\bar{F} \times \bar{C}) &= \bar{C} \cdot (\bar{\nabla} \times \bar{F}) - \bar{F} \cdot (\bar{\nabla} \times \bar{C}) = \bar{C} \cdot (\bar{\nabla} \times \bar{F}) \\ (\bar{F} \times \bar{C}) \cdot d\bar{s} &= -(\bar{F} \times d\bar{s}) \cdot \bar{C} \end{aligned}$$

$$\begin{aligned} \text{Thus, } \bar{C} \cdot \int_V (\bar{\nabla} \times \bar{F}) dv &= -\bar{C} \cdot \oint_S \bar{F} \times d\bar{s} \\ \longrightarrow \int_V (\bar{\nabla} \times \bar{F}) dv &= -\oint_S \bar{F} \times d\bar{s} \text{ because } \bar{C} \text{ is an arbitrary constant vector.} \end{aligned}$$

$$\text{P.6-15 a) } \left. \begin{aligned} \bar{H} &= \bar{a}_z n I \\ \bar{B} &= \bar{a}_z \mu n I \\ \bar{M} &= \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z \left(\frac{\mu}{\mu_0} - 1 \right) n I \end{aligned} \right\} \text{Eq. (6-13)} \quad \left| \begin{aligned} a < r < b: \bar{H} &= \bar{a}_z n I \\ \bar{B} &= \bar{a}_z \mu_0 n I \\ \bar{M} &= 0 \end{aligned} \right.$$

$$\text{b) } \bar{J}_m = \bar{\nabla} \times \bar{M} = 0; \quad \bar{J}_m = \bar{M} \times \bar{a}_n = (\bar{a}_z \times \bar{a}_r) \left(\frac{\mu}{\mu_0} - 1 \right) n I = \bar{a}_\phi \left(\frac{\mu}{\mu_0} - 1 \right) n I$$

P.6-17



$$\text{a) } V_m = \frac{I}{4\pi} \int \frac{d\bar{s} \cdot \bar{a}_z}{R^2} = \frac{I}{4\pi} \Omega,$$

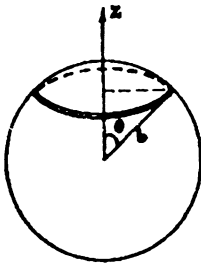
$$\begin{aligned} d\bar{s} \cdot \bar{a}_z &= (\cos \alpha) \rho d\rho d\phi \\ &= \frac{z}{\sqrt{z^2 + \rho^2}} \rho d\rho d\phi, \\ R &= \sqrt{z^2 + \rho^2}. \end{aligned}$$

$$\begin{aligned} \therefore V_m &= \frac{I}{4\pi} \int_0^{2\pi} \int_0^b \frac{z}{(z^2 + \rho^2)^{3/2}} \rho d\rho d\phi \\ &= \frac{I}{2} \left(1 - \frac{z}{\sqrt{z^2 + b^2}} \right). \end{aligned}$$

$$\text{b) } \bar{B} = -\mu_0 \bar{\nabla} V_m = -\bar{a}_z \mu_0 \frac{\partial V_m}{\partial z} = \bar{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}},$$

which is the same as Eq. (6-38).

P.6-18



$$\vec{M} = \vec{a}_z M_0$$

a) $\vec{J}_m = \vec{\nabla} \times \vec{M} = 0.$

$$\begin{aligned} \vec{J}_{ms} &= \vec{M} \times \vec{a}_n = \vec{a}_z M_0 \times \frac{1}{b} (\vec{a}_x x + \vec{a}_y y + \vec{a}_z z) \\ &= \frac{M_0}{b} (-\vec{a}_x y + \vec{a}_y x) - \vec{a}_\phi \frac{M_0}{b} \sqrt{x^2 + y^2} \\ &= \vec{a}_\phi M_0 \sin \theta. \end{aligned}$$

(Or, $\vec{J}_{ms} = (\vec{a}_x \cos - \vec{a}_y \sin) M_0 \times \vec{a}_x$
 $= \vec{a}_\phi M_0 \sin \theta$;

b) Apply Eq. (6-38) to a loop of radius $b \sin \theta$ carrying a current $J_{ms} b d\theta$:

$$d\vec{B} = \vec{a}_z \frac{\mu_0 (J_{ms} b d\theta) (b \sin \theta)^2}{2 (b^2)^{3/2}}$$

$$\begin{aligned} \vec{B} &= \int d\vec{B} = \vec{a}_z \frac{\mu_0 M_0}{2} \int_0^\pi \sin^3 \theta d\theta = \vec{a}_z \frac{\mu_0 M_0}{2} \sin^2 \theta \\ &= \vec{a}_z \frac{2}{3} \mu_0 M_0 = \frac{2}{3} \mu_0 \vec{M}. \end{aligned}$$

P.6-19 a) $\alpha_g = \frac{I_g}{\mu_0 S} = \frac{3 \times 10^3}{4\pi \times 10^{-7} \times \pi \times 0.025^2} = 1.21 \times 10^6 \text{ (H}^{-1}\text{)}$

$$\alpha_c = \frac{2\pi \times 0.02 - 0.003}{3000 \times (4\pi \times 10^{-7}) \times \pi \times 0.025^2} = 6.75 \times 10^4 \text{ (H}^{-1}\text{)}$$

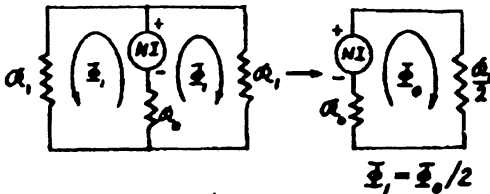
b) $\vec{B}_g = \vec{B}_c = \vec{a}_\phi \frac{10^{-5}}{\pi \times 0.025^2} = \vec{a}_\phi 5.09 \times 10^3 \text{ (T)}$

$$\vec{H}_g = \frac{1}{\mu_0} \vec{B}_g = \vec{a}_\phi \frac{5.09 \times 10^3}{4\pi \times 10^{-7}} = \vec{a}_\phi 4.05 \times 10^3 \text{ (A/m)}$$

$$\vec{H}_c = \frac{1}{\mu_0 \mu_r} \vec{B}_c = \vec{a}_\phi \frac{4.05 \times 10^3}{3000} = \vec{a}_\phi 1.35 \text{ (A/m)}$$

c) $N I = \oint (\alpha_c + \alpha_g), \quad I = \frac{1}{N} \oint (\alpha_c + \alpha_g) = \frac{10^5}{500} \times 1.2775 \times 10^6$
 $= 0.0256 \text{ (A)} = 25.6 \text{ (mA)}$.

P.6-20 Magnetic circuit:



$$S = 10^{-3} \text{ (m}^2\text{)}$$

$$\frac{1}{\mu_0 S} = \frac{1}{(4\pi \times 10^{-7}) \times 10^{-3}} = 7.95 \times 10^4$$

Neglecting leakage flux and assuming constant flux density over S ,

$$\alpha_0 = \frac{1}{\mu_0 S} (0.002 - \frac{0.24 - 0.02}{3000}) = 1.60 \times 10^6 \text{ (H}^{-1}\text{)}$$

$$\alpha_1 = \frac{1}{\mu_0 S} (\frac{0.24 + 2 \times 0.2}{3000}) = 0.102 \times 10^6 \text{ (H}^{-1}\text{)}$$

$$\Phi_0 = \frac{NI}{a_0 + a_1/2} = \frac{200 \times 3}{(1.60 + \frac{0.02}{2}) \times 10^2} = 3.63 \times 10^{-4} \text{ (Wb)}$$

$$\Phi_1 = \frac{\Phi_0}{2} = 1.82 \times 10^{-4} \text{ (Wb)}$$

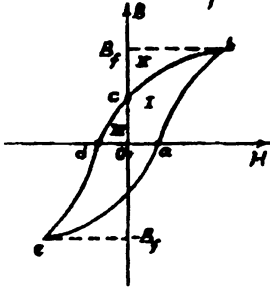
$$b) H_1 = \frac{B_1}{\mu_0 \mu_r} = \frac{\Phi_1}{\mu_0 \mu_r S} = (7.95 \times 10^8) \frac{1.82 \times 10^{-4}}{5000} = 28.9 \text{ (A/m)}$$

$$(H_0)_g = \frac{B_0}{\mu_0} = \frac{1}{\mu_0 S} \Phi_0 = (7.95 \times 10^8) \times 3.63 \times 10^{-4} = 28.9 \times 10^4 \text{ (A/m)},$$

in air gap.

$$(H_0)_c = (H_0)_g / 5000 = 57.8 \text{ (A/m)}, \text{ in core.}$$

6-21 a) Work required per unit length in time dt:



$$P_1 dt = n I d\Phi$$

Work per unit volume in dt:

$$dW = \frac{1}{S} P_1 dt = n I dB = H dB$$

$$\text{Thus, } W_1 = \int_0^{B_f} H dB$$

b) Work done per unit volume in changing from 0 to B_f along path ab is W_1 , which is represented by areas I and II.

Along path bc, B is decreased, inducing a voltage that tends to maintain the current. Work is done against the source. The work per unit volume W_2 is represented by $-(\text{area II})$. In going from c to d, the direction of current is reversed and the work done W_3 is represented by area III. Same amount of work is done in changing B along the path from d to e and back to a as that required in going from a to b through c to d.

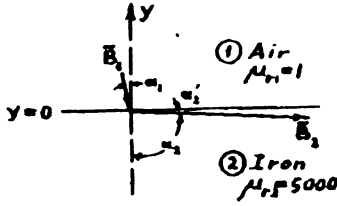
\therefore Work done per unit volume in one cycle = $2(W_1 + W_2 + W_3)$
 $= 2 \times \text{Areas}[(\text{I} + \text{II}) - \text{II} + \text{III}] = \text{Area of the hysteresis loop}$

$$6-23 \quad \vec{H}_1 = -\vec{\nabla} V_{m1}, \quad \vec{H}_2 = -\vec{\nabla} V_{m2}$$

$$\text{Boundary Conditions: } \mu_1 H_{1n} = \mu_2 H_{2n} \longrightarrow \mu_1 \frac{\partial V_{m1}}{\partial n} = \mu_2 \frac{\partial V_{m2}}{\partial n}$$

$$H_{1t} - H_{2t} = J_{sn} \longrightarrow \frac{\partial V_{m2}}{\partial t} - \frac{\partial V_{m1}}{\partial t} = J_{sn}$$

P.6-24



$$a) \bar{B}_1 = \bar{a}_x 0.5 - \bar{a}_y 10 \text{ (mT)}$$

$$\bar{B}_2 = \bar{a}_x B_{2x} + \bar{a}_y B_{2y}$$

$$H_{2x} = \frac{B_{2x}}{5000\mu_0} = H_{1x} = \frac{0.5}{\mu_0}$$

$$\longrightarrow B_{2x} = 2,500 \text{ (mT)}$$

$$B_{2y} = B_{1y} = -10 \text{ (mT)}$$

$$\therefore \bar{B}_2 = \bar{a}_x 2500 - \bar{a}_y 10 \text{ (mT)}$$

$$\tan \alpha_2 = \frac{\mu_2}{\mu_1} \tan \alpha_1$$

$$= 5000 \tan 0.05 \approx 250 \longrightarrow \alpha_2 = 89.77^\circ, \alpha_2' = 0.23^\circ$$

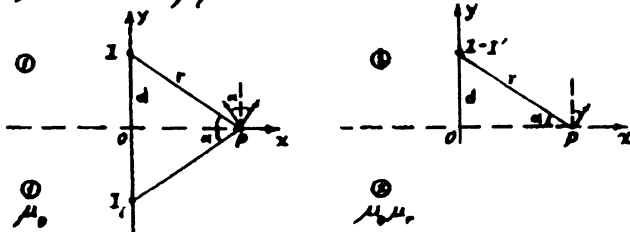
$$b) \text{ If } \bar{B}_2 = \bar{a}_x 10 + \bar{a}_y 0.5 \text{ (mT)}, \bar{B}_1 = \bar{a}_x B_{1x} + \bar{a}_y B_{1y}$$

$$H_{1x} = \frac{B_{1x}}{\mu_1} = H_{2x} = \frac{B_{2x}}{\mu_2} \longrightarrow B_{1x} = \frac{\mu_{r1}}{\mu_{r2}} B_{2x} = \frac{10}{5000} = 0.002$$

$$B_{1y} = B_{2y} = 0.5 \quad \therefore \bar{B}_1 = \bar{a}_x 0.002 + \bar{a}_y 0.5 \text{ (mT)}$$

$$\alpha_1 = \tan^{-1} \frac{B_{1x}}{B_{1y}} \approx \frac{0.002}{0.5} = 0.004 \text{ (rad)} = 0.23^\circ$$

P.6-25 a) Consider two situations: ① I and I_i both in air; and ② I and $-I$ both in magnetic medium with relative permeability μ_r .



Find B_{1y} and H_{1x} at $P(y=0)$. Find B_{2y} and H_{2x} at $P(y=0)$.

$$B_{1y} = \frac{\mu_0}{2\pi r} (I + I_i) \cos \alpha = \frac{\mu_0 \mu_r}{\pi(\mu_r + 1)} \frac{x}{r^2} I; \quad B_{2y} = \frac{\mu_0 \mu_r}{2\pi r} (I - I_i) \cos \alpha = \frac{\mu_0 \mu_r}{\pi(\mu_r + 1)} \frac{x}{r^2} I$$

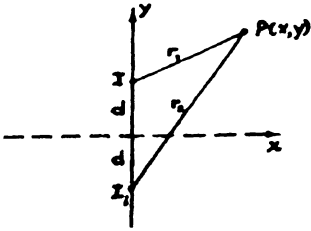
$$B_{1x} = \frac{\mu_0}{2\pi r} (I - I_i) \sin \alpha = \frac{\mu_0}{\pi(\mu_r + 1)} \frac{d}{r^2} I; \quad B_{2x} = \frac{\mu_0 \mu_r}{2\pi r} (I - I_i) \sin \alpha = \frac{\mu_0 \mu_r}{\pi(\mu_r + 1)} \frac{d}{r^2} I$$

$$H_{1x} = \frac{B_{1x}}{\mu_0} = \frac{I}{\pi(\mu_r + 1)} \frac{d}{r^2}; \quad H_{2x} = \frac{B_{2x}}{\mu_r \mu_0} = \frac{I}{\pi(\mu_r + 1)} \frac{d}{r^2}$$

$$\therefore B_{1y} = B_{2y} \text{ and } H_{1x} = H_{2x} \text{ (Boundary conditions satisfied)}$$

$$b) \text{ For } \mu_r \gg 1, I_i = \frac{\mu_r - 1}{\mu_r + 1} I \approx I$$

Refer to following figure.

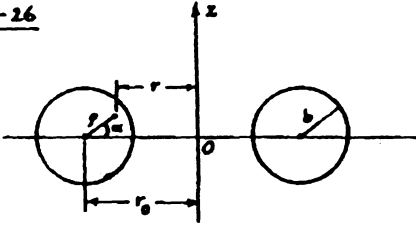


$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r_1} (-\vec{a}_x \frac{y-d}{r_1} + \vec{a}_y \frac{x}{r_1})$$

$$\vec{B}_{I_1} = \frac{\mu_0 I}{2\pi r_2} (-\vec{a}_x \frac{y+d}{r_2} + \vec{a}_y \frac{x}{r_2})$$

$$\begin{aligned} \therefore \vec{B} &= \vec{B}_1 + \vec{B}_{I_1} \\ &= -\vec{a}_x \frac{\mu_0 I}{2\pi} \left[\frac{y-d}{(y-d)^2+x^2} + \frac{y+d}{(y+d)^2+x^2} \right] \\ &\quad + \vec{a}_y \frac{\mu_0 I x}{2\pi} \left[\frac{1}{(y-d)^2+x^2} + \frac{1}{(y+d)^2+x^2} \right] \end{aligned}$$

P.6-26



$$\vec{B} = \vec{a}_\phi B_\phi = \vec{a}_\phi \frac{\mu_0 N I}{2\pi r}$$

$$r = r_0 - \rho \cos \alpha$$

$$\begin{aligned} \vec{\Phi} &= \frac{\mu_0 N I}{2\pi} \int_0^b \int_0^{2\pi} \frac{\rho d\alpha d\rho}{r_0 - \rho \cos \alpha} \\ &= \frac{\mu_0 N I}{2\pi} \int_0^b \left[\frac{2\pi}{\sqrt{r_0^2 - \rho^2}} \right] \rho d\rho \\ &= \mu_0 N I (r_0 - \sqrt{r_0^2 - b^2}) \end{aligned}$$

$$\therefore L = \frac{N \vec{\Phi}}{I} = \mu_0 N^2 (r_0 - \sqrt{r_0^2 - b^2})$$

If $r_0 \gg b$, $B_\phi \approx \frac{\mu_0 N I}{2\pi r_0}$ (constant).

$$\vec{\Phi} \approx B_\phi S = \frac{\mu_0 N I}{2\pi r_0} \cdot \pi b^2 = \frac{\mu_0 N b^2 I}{2 r_0}$$

$$L = \frac{N \vec{\Phi}}{I} \approx \frac{\mu_0 N^2 b^2}{2 r_0}$$

P.6-27 For $b \leq r \leq (b+d)$, $\vec{B}_3 = \vec{a}_\phi B_{\phi 3} = \vec{a}_\phi \frac{\mu_0 I}{2\pi r} \left[1 - \frac{\pi(r^2 - b^2)}{\pi(b+d)^2 - \pi b^2} \right]$
 $= \vec{a}_\phi \frac{\mu_0 I}{2\pi r} \left[\frac{(b+d)^2 - r^2}{(b+d)^2 - b^2} \right]$

Magnetic energy per unit length stored in the outer conductor

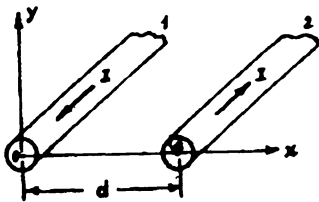
$$\begin{aligned} W'_{m3} &= \frac{1}{2\mu_0} \int_b^{b+d} B_{\phi 3}^2 2\pi r dr \\ &= \frac{\mu_0 I^2}{4\pi} \left\{ \frac{(b+d)^4}{[(b+d)^2 - b^2]^2} \ln\left(1 + \frac{d}{b}\right) + \frac{b^2 - 3(b+d)^2}{4[(b+d)^2 - b^2]} \right\} \end{aligned}$$

From Eqs. (6-154) - (6-155b) on p. 246 we have

$$\begin{aligned} L' &= \frac{2}{I^2} (W'_{m1} + W'_{m2} + W'_{m3}) \\ &= \frac{\mu_0}{2\pi} \left\{ \frac{1}{4} + \ln \frac{b}{a} - \frac{(b+d)^4}{[(b+d)^2 - b^2]^2} \ln\left(1 + \frac{d}{b}\right) + \frac{b^2 - 3(b+d)^2}{4[(b+d)^2 - b^2]} \right\} \end{aligned}$$

(H/m).

P.6-28



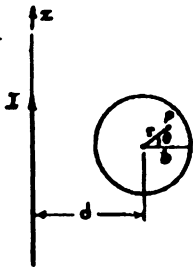
$$\begin{aligned}\Phi'_e &= \int_a^{d-a} (B_{y1} + B_{y2}) dx \\ &= \int_a^{d-a} \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)} \right] dx \\ &= \frac{\mu_0 I}{\pi} \ln \left(\frac{d-a}{a} \right) \approx \frac{\mu_0 I}{\pi} \ln \frac{d}{a} \\ \therefore L'_e &= \frac{\Phi'_e}{I} \approx \frac{\mu_0}{\pi} \ln \frac{d}{a} \quad (\text{H/m})\end{aligned}$$

$$L' = L'_i + L'_e = \frac{\mu_0}{4\pi} + \frac{\mu_0}{\pi} \ln \frac{d}{a} \quad (\text{H/m}).$$

P.6-29 For a current I in the long straight wire,

$$\begin{aligned}\bar{B} &= \bar{a}_\phi \frac{\mu_0 I}{2\pi r} \\ \Lambda_{12} &= \int_S \bar{B} \cdot d\bar{s} = 2 \int B_\phi \frac{1}{\sqrt{3}} (r-d) dr = \frac{\mu_0 I}{\pi \sqrt{3}} \int_d^{d+\sqrt{3}b} \left(\frac{r-b}{r} \right) dr \\ &= \frac{\mu_0 I}{\pi \sqrt{3}} \left[\frac{\sqrt{3}}{2} b - d \ln \left(1 + \frac{\sqrt{3}b}{2d} \right) \right] \\ L_{12} &= \frac{\Lambda_{12}}{I} = \frac{\mu_0}{\pi} \left[\frac{b}{2} - \frac{d}{\sqrt{3}} \ln \left(1 + \frac{\sqrt{3}b}{2d} \right) \right] \quad (\text{H}).\end{aligned}$$

P.6-30



Assume a current I .

$$\begin{aligned}\bar{B} \text{ at } P(r, \theta) &\text{ is } \bar{a}_\phi \frac{\mu_0 I}{2\pi(d+r\cos\theta)} \\ \Lambda_{12} &= \frac{\mu_0 I}{2\pi} \int_0^{2\pi} \int_0^b \frac{r dr d\theta}{d+r\cos\theta} = \frac{\mu_0 I}{2\pi} \int_0^{2\pi} \int_0^b \frac{2\pi r dr}{\sqrt{d^2-r^2}} \\ &= \mu_0 I (d - \sqrt{d^2 - b^2}). \\ L_{12} &= \mu_0 (d - \sqrt{d^2 - b^2}).\end{aligned}$$

P.6-31 Since $h_1 \gg h_2$, the magnetic flux due to the long loop linking with the small loop can be approximated by that due to two infinitely long wires carrying equal and opposite current I .

$$\begin{aligned}\Lambda_{12} &= \frac{\mu_0 I}{2\pi} \int_0^{w_2} \left(\frac{1}{d+x} - \frac{1}{w_1+d+x} \right) dx = \frac{\mu_0 I}{2\pi} \ln \left(\frac{w_2+d}{d} \cdot \frac{w_1+d}{w_1+w_2+d} \right) \\ L &= \frac{\Lambda_{12}}{I} = \frac{\mu_0}{2\pi} \ln \frac{(w_1+d)(w_2+d)}{d(w_1+w_2+d)}.\end{aligned}$$

P.6-32 Eq. (6-140): $W_2 = \frac{1}{2} L_1 I_1^2 + L_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$.

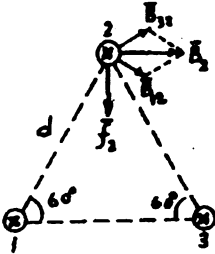
$$\begin{aligned}\text{a) } W_2 &= \frac{I_2^2}{2} \left[L_1 \left(\frac{I_1}{I_2} \right)^2 + 2L_{12} \left(\frac{I_1}{I_2} \right) + L_2 \right] \\ &= \frac{I_2^2}{2} (L_1 x^2 + 2L_{12} x + L_2), \quad x = \frac{I_1}{I_2}.\end{aligned}$$

$$\frac{dW_2}{dx} = \frac{I_1^2}{2} (2L_1 x + 2L_{21}) = 0, \quad \frac{d^2W_2}{dx^2} = I_1^2 L_1 > 0.$$

$$\therefore x = -\frac{I_1}{I_2} = -\frac{L_{21}}{L_1} \text{ for minimum } W_2.$$

$$b) (W_2)_{\min} = \frac{I_1^2}{2} (L_2 - \frac{L_{21}^2}{L_1}) \geq 0 \longrightarrow L_{21} \leq \sqrt{L_1 L_2}.$$

P.6-33



$$I_1 = I_2 = I_3 = 25 \text{ (A)}$$

$$d = 0.15 \text{ (m)}$$

$$\begin{aligned} \vec{B}_2 &= \vec{a}_x 2B_{12} \cos 30^\circ = \vec{a}_x 2 \left(\frac{\mu_0 I}{2\pi d} \right) \frac{\sqrt{3}}{2} \\ &= \vec{a}_x \frac{\mu_0 I \sqrt{3}}{2\pi d}. \end{aligned}$$

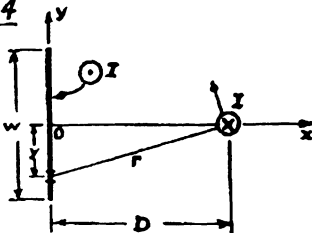
Force per unit length on wire 2:

$$\vec{f}_2 = -\vec{a}_y I B_2 = -\vec{a}_y \frac{\mu_0 I^2 \sqrt{3}}{2\pi d}$$

$$= -\vec{a}_y 1150 \mu_0 = -\vec{a}_y 1.44 \times 10^{-3} \text{ (N/m)}.$$

Forces on all three wires are of equal magnitude and toward the center of the triangle.

P.6-34



$$\text{Elemental strip } dy: dI = \frac{I}{w} dy,$$

$$|d\vec{H}| = \frac{dI}{2\pi r} = \frac{I dy}{2\pi w \sqrt{D^2 + y^2}}.$$

Symmetry $\rightarrow \vec{H}$ has only a y-component.

$$\begin{aligned} \vec{H} \text{ (at wire)} &= \vec{a}_y \int dH \cdot \left(\frac{D}{r} \right) \\ &= \vec{a}_y 2 \int_0^{w/2} \frac{ID}{2\pi w \sqrt{D^2 + y^2}} dy \\ &= \vec{a}_y \frac{I}{\pi w} \tan^{-1} \left(\frac{w}{2D} \right). \end{aligned}$$

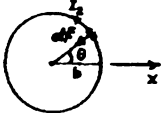
$$\vec{f} = \vec{I} \times \vec{B}$$

$$= (-\vec{a}_x I) \times (\mu_0 \vec{H}) = \vec{a}_x \frac{\mu_0 I^2}{\pi w} \tan^{-1} \left(\frac{w}{2D} \right) \text{ (N/m)}.$$

P.6-35

\vec{B} due I_1 in straight wire in z direction at an elemental arc $b d\theta$ on the circular loop:

$$\vec{B} = \vec{a}_\phi \frac{\mu_0 I_1}{2\pi (d + b \cos \theta)} \quad \text{Vertical component of } d\vec{F} \text{ on } b d\theta \text{ at } \theta \text{ is cancelled by that on } b d\theta \text{ at } -\theta$$



$$\therefore \vec{F} = -\vec{a}_x 2 \int_0^\pi (I_1 b d\theta) B \cos \theta \text{ on loop}$$

$$\begin{aligned} \vec{F} &= -\vec{a}_x \frac{\mu_0 I_1 I_2 b}{\pi} \int_0^\pi \frac{\cos \theta d\theta}{d + b \cos \theta} \\ &= a_x \mu_0 I_1 I_2 \left[\frac{1}{\sqrt{1 - (b/d)^2}} - 1 \right] \quad (\text{Repulsive force}) \end{aligned}$$

P.6-36 Resolve the circular loop into many small loops, each with a magnetic dipole moment $d\vec{m} = I_2 d\vec{s}$.

$$\vec{T} = \int d\vec{T} = I_2 \int d\vec{s} \times \vec{B} \quad d\vec{T} = d\vec{m} \times \vec{B}$$

$$= -\vec{a}_x I_2 \sin \alpha \int B ds = -\vec{a}_x \mu_0 I_1 I_2 (d - \sqrt{d^2 - b^2}) \sin \alpha$$

in the direction of aligning the direction of the flux by I_2 in the loop to that of \vec{B} due to I_1 in the straight wire.

($\int B ds$ over the circular loop has been found in problem P.6-30 as Λ_{12} .)

P.6-37 Magnetic flux density at the center of the large circular turn of wire carrying current I_2 is

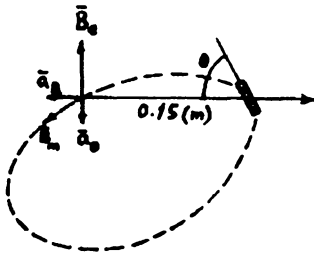
$$\vec{B}_2 = \vec{a}_{z2} \frac{\mu_0 I_2}{2 r_2} \quad (\text{Set } z=0 \text{ in Eq. (6-38)})$$

Torque on the small circular wire:

$$\vec{T} = \vec{m} \times \vec{B}_2 \approx (\vec{a}_{z1} I_1 \pi r_1^2) \times (\vec{a}_{z2} \frac{\mu_0 I_2}{2 r_2}) = (\vec{a}_{z1} \times \vec{a}_{z2}) \frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2}$$

→ Magnitude = $\frac{\mu_0 I_1 I_2 \pi r_1^2}{2 r_2} \sin \alpha$, in a direction to align the magnetic fluxes produced by I_1 and I_2 .

P.6-38



\vec{B}_m (magnetized compass needle)

$$\begin{aligned} &= \frac{\mu_0 m}{4 \pi R^3} (\vec{a}_x 2 \cos \theta + \vec{a}_y \sin \theta) \\ &= \frac{(4 \pi \times 10^{-7}) \times 2}{4 \pi (0.15)^3} (\vec{a}_x 2 \cos \theta + \vec{a}_y \sin \theta) \\ &= \frac{16}{27} \times 10^{-4} (\vec{a}_x 2 \cos \theta + \vec{a}_y \sin \theta) \quad (T) \end{aligned}$$

$$\vec{B}_e \text{ (earth)} = -\vec{a}_y 10^{-4} \text{ (T)}$$

Max. deflection when $|B_x/B_y|$ is max., or when

$$\left| \frac{B_x}{B_y} \right| = \left| \frac{(\frac{16}{27} \sin \theta - 1) \times 10^{-4}}{\frac{16}{27} \times 10^{-4} \cos \theta} \right| \text{ is min.}$$

$$\text{Set } \frac{d}{d\theta} \left(\frac{1 - \frac{16}{27} \sin \theta}{\frac{16}{27} \cos \theta} \right) = 0 \rightarrow \sin \theta = \frac{16}{27}, \text{ or } \theta = 36.34^\circ$$

$$\text{At } \theta = 36.34^\circ, |B_x/B_y| = 1.471 \text{ and } \alpha = \tan^{-1} 1.471 = 55.8^\circ$$

(If the bar magnet is oriented such that $\vec{B}_m \perp \vec{B}_e$, then $\alpha = 49.8^\circ < 55.8^\circ$.)

$$P.6-39 \quad F = \frac{\vec{Q}^2}{\mu_0 S} = \frac{(NI)^2}{\mu_0 S \left(\frac{2\ell_2}{\mu_0 S} - \frac{\ell_1}{\mu_0 \mu_r S} \right)^2} = \frac{(NI)^2 \mu_0 S}{(2\ell_2 - \frac{\ell_1}{\mu_r})^2}$$

$$F = 100 \times 9.8 = 980 \text{ (N)}, \quad S = 0.01 \text{ (m}^2\text{)}, \quad \ell_2 = 2 \times 10^{-3} \text{ (m)}, \\ \ell_1 = 3 \text{ (m)}, \quad \mu_r = 4000.$$

$$\text{Solving: } mmf = NI = 1.326 \times 10^3 \text{ (A}\cdot\text{t)}$$

$$P.6-40 \quad W_m = \frac{1}{2} \int \mu H^2 dv$$

Assume a virtual displacement, Δx , of the iron core.

$$W_m(x + \Delta x) = W_m(x) + \frac{1}{2} \int_{S \Delta x} (\mu - \mu_0) H^2 dv \\ = W_m(x) + \frac{1}{2} \mu_0 (\mu_r - 1) n^2 I^2 S \Delta x$$

$$(F_x)_x = \frac{\partial W_m}{\partial x} = \frac{\mu_0}{2} (\mu_r - 1) n^2 I^2 S, \text{ in the direction of increasing } x.$$

Chapter 7

$$P.7-1 \quad \mathcal{V} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = - \int_S \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} = - \oint_C \frac{\partial \vec{A}}{\partial t} \cdot d\vec{L}.$$

$$P.7-2 \quad \vec{B} = \vec{a}_z 3 \cos(5\pi 10^7 t - \frac{2}{3}\pi x) \cdot 10^{-6} \text{ (T)}$$

$$\int_S \vec{B} \cdot d\vec{s} = \int_0^{0.6} \vec{a}_z 3 \cos(5\pi 10^7 t - \frac{2}{3}\pi x) 10^{-6} \cdot (\vec{a}_z 0.2 dx) \\ = - \frac{0.18}{2\pi} [\sin(5\pi 10^7 t - 0.4\pi) - \sin 5\pi 10^7 t] \cdot 10^{-6} \text{ (Wb)}$$

$$\mathcal{V} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s} = 45 [\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t] \text{ (V)}$$

$$i = \frac{\mathcal{V}}{2R} = 1.5 [\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t]$$

$$= -3 \sin(5\pi 10^7 t - 0.2\pi) \sin(-0.2\pi)$$

$$= 1.76 \sin(5\pi 10^7 t - 0.2\pi) \text{ (A)}$$

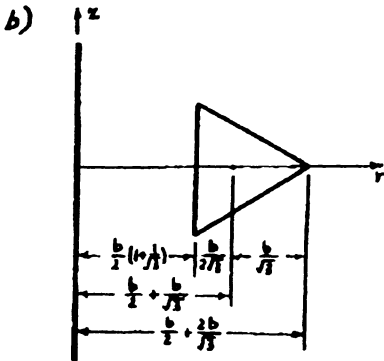
$$P.7-3 \quad \vec{B} = \vec{a}_\phi \frac{\mu_0 I \sin \omega t}{2\pi r}. \quad \vec{\mathcal{E}} = \int_S \vec{B} \cdot d\vec{s}, \quad ds = \vec{a}_\rho 2\pi r dr, \quad z = \frac{\sqrt{3}}{3}(r-d)$$

$$a) \quad \vec{\mathcal{E}} = \frac{\sqrt{3}}{3} \frac{\mu_0 I \sin \omega t}{\pi} \int_d^{\sqrt{3}b+d} \left(1 - \frac{d}{r}\right) dr = \frac{\sqrt{3} \mu_0 I \sin \omega t}{3\pi} \left[\frac{\sqrt{3}}{2} b - d \ln\left(\frac{\sqrt{3}b+d}{d}\right) \right]$$

$$d = \frac{b}{2}, \quad \mathcal{V} = - \frac{\partial \vec{\mathcal{E}}}{\partial t} = - \frac{\sqrt{3} \mu_0 I \omega b}{3\pi} \left[\frac{\sqrt{3}}{2} - \frac{1}{2} \ln(\sqrt{3}+1) \right] \cos \omega t \\ = V_m \cos \omega t$$

$$V_{rms} = \frac{\sqrt{2}}{2} |V_m| = \frac{\sqrt{6} \mu_0 I \omega b}{12 \pi} [\sqrt{3} - \ln(\sqrt{3} + 1)]$$

$$= 0.0472 \mu_0 I \omega b \text{ (V)}$$



$$z = \frac{1}{\sqrt{3}} \left[\frac{b}{2} \left(1 + \frac{4}{\sqrt{3}} \right) - r \right]$$

$$\int \vec{B} \cdot d\vec{s} = \frac{\mu_0 I \sin \omega t}{\sqrt{3} \pi} \int_{\frac{b}{2}(1+\frac{4}{\sqrt{3}})}^{\frac{b}{2}(1+\frac{4}{\sqrt{3}})} \left[\frac{b}{2} \left(1 + \frac{4}{\sqrt{3}} \right) \frac{1}{r} - 1 \right] dr$$

$$= \frac{\mu_0 I \sin \omega t}{\sqrt{3} \pi} \left[\frac{b}{2} \left(1 + \frac{4}{\sqrt{3}} \right) \ln \left(\frac{4+\sqrt{3}}{1+\sqrt{3}} \right) - \frac{\sqrt{3}}{2} b \right]$$

$$V_{rms} = \frac{1}{\sqrt{2}} \frac{\mu_0 I \omega}{\sqrt{3} \pi} \frac{b}{2} \left| \left(1 + \frac{4}{\sqrt{3}} \right) \ln \left(\frac{4+\sqrt{3}}{1+\sqrt{3}} \right) - \sqrt{3} \right|$$

$$= 0.0469 \mu_0 I \omega b \text{ (V)}$$

P.7-4 From problem P.6-30: $\vec{\Phi}_{12} = \mu_0 I (\sin \omega t) (d - \sqrt{d^2 - b^2})$

a)

$$-V = -\frac{d\Phi}{dt} = -\mu_0 I \omega (\cos \omega t) (d - \sqrt{d^2 - b^2})$$

$$= V_m \cos \omega t$$

$$I_{rms} = \frac{|V_m|}{\sqrt{2} R} = \frac{\mu_0 I \omega (d - \sqrt{d^2 - b^2})}{\sqrt{2} R}$$

$$I = \frac{\sqrt{2} R \times 3 \times 10^{-9}}{\mu_0 \omega (d - \sqrt{d^2 - b^2})} = \frac{3\sqrt{2} \times 10^{-6}}{4\pi \times 10^{-7} (2\pi 60) \times 0.0382} = 0.234 \text{ (A)}$$

b) $\alpha = \cos^{-1} \left(\frac{0.2}{0.3} \right) = 48.2^\circ$

P.7-5 $\vec{\Phi}(t) = B(t)S(t) = (5 \cos \omega t) \times 0.2 (0.7 - x)$

$$= 0.35 \cos \omega t (1 + \cos \omega t) \text{ (mT)}$$

$$i = -\frac{1}{R} \frac{d\Phi}{dt} = -\frac{1}{R} 0.35 \omega (\sin \omega t + \sin 2\omega t)$$

$$= -1.75 \omega \sin \omega t (1 + 2 \cos \omega t) \text{ (mA)}$$

P.7-6

$$i = \frac{V}{R} = -\frac{1}{R} \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s} = -\frac{1}{R} \frac{d}{dt} (B_0 h w \cos \omega t)$$

$$= \frac{\omega B_0 h w}{R} \sin \omega t$$

$$P_d = \gamma i = \frac{(\omega B_0 h w)^2}{R} \sin^2 \omega t \text{ (Power dissipated in } R)$$

On the other hand, for side 1-2: $\vec{F}_{12} = \vec{a}_2 \times h \vec{B}_0$, $\vec{u}_{12} = -\vec{a}_2 \frac{\omega w}{2} \sin \omega t$

for side 4-3: $\vec{F}_{43} = -\vec{a}_2 \times h \vec{B}_0$, $\vec{u}_{43} = \vec{a}_2 \frac{\omega w}{2} \sin \omega t$

Mechanical power required to rotate coil $P_m = -(\vec{F}_{12} \cdot \vec{u}_{12} + \vec{F}_{43} \cdot \vec{u}_{43}) = -(\omega B_0 h w i) = P_d$

7-7 Take the divergence of Eq. (7-37a):

$$\bar{\nabla} \cdot (\bar{\nabla} \times \bar{E}) = -\frac{\partial}{\partial t} (\bar{\nabla} \cdot \bar{B}) = 0 \quad \text{from Eq. (2-137)}$$

→ $\bar{\nabla} \cdot \bar{B} = f(x, y, z)$, which is to hold at all times everywhere whether \bar{B} exists or not; hence $f(x, y, z)$ must vanish and $\bar{\nabla} \cdot \bar{B} = 0$.

Similarly, take the divergence of Eq. (7-37b):

$$\begin{aligned} \bar{\nabla} \cdot (\bar{\nabla} \times \bar{H}) &= \bar{\nabla} \cdot \bar{J} + \frac{\partial}{\partial t} (\bar{\nabla} \cdot \bar{D}) = 0 \\ &= -\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial t} (\bar{\nabla} \cdot \bar{D}), \quad \text{from Eq. (7-32)} \end{aligned}$$

$$\longrightarrow \bar{\nabla} \cdot \bar{D} = \rho.$$

7-8 Eq. (7-46): $\bar{\nabla} \cdot \bar{A} + \mu \epsilon \frac{\partial V}{\partial t} = 0.$

$$\bar{A} = \frac{\mu}{4\pi} \int_V \frac{\bar{J}}{R} dv', \quad V = \frac{1}{4\pi\epsilon} \int_V \frac{\rho}{R} dv'.$$

$$\therefore \frac{\mu}{4\pi} \left\{ \int_V \left[\bar{\nabla} \cdot \left(\frac{\bar{J}}{R} \right) + \frac{1}{R} \frac{\partial \rho}{\partial t} \right] dv' \right\} = 0. \quad (i)$$

$$\begin{aligned} \text{Now, } \bar{\nabla} \cdot \left(\frac{\bar{J}}{R} \right) &= \frac{1}{R} \bar{\nabla} \cdot \bar{J} + \bar{J} \cdot \bar{\nabla} \left(\frac{1}{R} \right) = \bar{J} \cdot \bar{\nabla} \left(\frac{1}{R} \right) \\ &= -\bar{J} \cdot \bar{\nabla}' \left(\frac{1}{R} \right). \end{aligned} \quad (ii)$$

$$\text{Also, } \bar{\nabla}' \cdot \left(\frac{\bar{J}}{R} \right) = \frac{1}{R} \bar{\nabla}' \cdot \bar{J} + \bar{J} \cdot \bar{\nabla}' \left(\frac{1}{R} \right). \quad (iii)$$

$$\text{Substituting (iii) in (ii), } \bar{\nabla} \cdot \left(\frac{\bar{J}}{R} \right) = \frac{1}{R} \bar{\nabla}' \cdot \bar{J} - \bar{\nabla}' \cdot \left(\frac{\bar{J}}{R} \right). \quad (iv)$$

$$\begin{aligned} \text{Substituting (iv) in (i), } \int_V \frac{1}{R} \left(\bar{\nabla}' \cdot \bar{J} + \frac{\partial \rho}{\partial t} \right) dv' &= \int_V \bar{\nabla}' \cdot \left(\frac{\bar{J}}{R} \right) dv' \\ &= \oint_S \frac{\bar{J} \cdot d\bar{i}'}{R}. \end{aligned}$$

$$\text{Let } R \rightarrow \infty, S' \rightarrow \infty, \bar{J} \cdot d\bar{s} = 0.$$

$$\therefore \bar{\nabla}' \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0 \quad (\text{Eq. of continuity}).$$

7-9 Eq. (7-37b): $\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \longrightarrow \bar{\nabla} \times \left(\frac{\bar{E}}{\mu} \right) = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t}.$

$$\text{From Eqs. (7-39) and (7-41): } \bar{B} = \bar{\nabla} \times \bar{A}, \quad \bar{E} = -\bar{\nabla} V - \frac{\partial \bar{A}}{\partial t}.$$

$$\mu \bar{\nabla} \times \left(\frac{1}{\mu} \bar{\nabla} \times \bar{A} \right) = \mu \bar{J} - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2} - \mu \epsilon \bar{\nabla} \left(\frac{\partial V}{\partial t} \right).$$

Use gauge condition for potentials in an inhomogeneous medium:

$$\bar{\nabla} \cdot (\epsilon \bar{A}) + \mu \epsilon^2 \frac{\partial V}{\partial t} = 0$$

→ Wave equation for vector potential:

$$-\mu \bar{\nabla} \times \left(\frac{1}{\mu} \bar{\nabla} \times \bar{A} \right) + \mu \epsilon \bar{\nabla} \left[\frac{1}{\mu \epsilon} \bar{\nabla} \cdot (\epsilon \bar{A}) \right] - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J}.$$

$$\text{From Eq. (7-37c), } \bar{\nabla} \cdot \bar{D} = \rho \longrightarrow \bar{\nabla} \cdot (\epsilon \bar{\nabla} V) + \frac{\partial}{\partial t} \bar{\nabla} \cdot (\epsilon \bar{A}) = -\rho.$$

$$\text{Wave equation for scalar potential: } \frac{1}{\epsilon} \bar{\nabla} \cdot (\epsilon \bar{\nabla} V) - \mu \epsilon \frac{\partial^2 V}{\partial t^2} = -\rho.$$

P.7-12 As shown in problem P.7-7:

a) Eq. (7-37d) can be derived from Eq. (7-37a). Hence the boundary conditions for the normal components of \vec{B} , which are obtained from $\vec{\nabla} \cdot \vec{B} = 0$, are not independent of the boundary conditions for the tangential components of \vec{E} , which are obtained from $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

b) Similarly, the boundary conditions for the normal components of \vec{D} are not independent of those for the tangential components of \vec{H} in the time-varying case.

P.7-13 Medium 1: Free space; medium 2: $\mu_2 \rightarrow \infty$.

\vec{H}_2 must be zero so that \vec{B}_2 is not infinite.

$$\therefore \bar{a}_{n2} \times \vec{H}_1 = 0; \quad B_{n1} = B_{n2}$$

$$\bar{a}_{n2} \times (\vec{D}_1 - \vec{D}_2) = \rho_s; \quad E_{t1} = E_{t2}$$

P.7-14 $\vec{E}_1(z,t) = \bar{a}_x 0.03 \sin 10^8 \pi (t - \frac{z}{c}) = \bar{a}_x \text{Re} [0.03 e^{-j\pi/2} e^{j10^8 \pi (t-z/c)}]$

$$\vec{E}_2(z,t) = \bar{a}_x 0.04 \cos [10^8 \pi (t - \frac{z}{c}) - \frac{\pi}{3}] = \text{Re} [0.04 e^{j\pi/2} e^{j10^8 \pi (t-z/c)}]$$

Phasors: $\vec{E} = \vec{E}_1 + \vec{E}_2 = \bar{a}_x [0.03 e^{-j\pi/2} + 0.04 e^{-j\pi/3}]$

$$= \bar{a}_x [-j0.03 + (0.02 - j0.02\sqrt{3})]$$

$$= \bar{a}_x (0.068 e^{j1.27}) = \bar{a}_x E_0 e^{j\theta}$$

$$\therefore E_0 = 0.068, \quad \theta = -1.27 \text{ rad. or } -72.8^\circ$$

P.7-16 $\vec{\nabla}^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$ with $V(R,t) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(t-R/u)}{R} dv$

$$\vec{\nabla}^2 V - \frac{1}{u^2} \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$$

We need $\vec{\nabla}^2 (\frac{\rho}{R}) = \frac{1}{R} \vec{\nabla}^2 \rho + \rho \vec{\nabla}^2 (\frac{1}{R}) + 2(\vec{\nabla} \rho) \cdot \vec{\nabla} (\frac{1}{R})$.

{Formula: $\vec{\nabla}^2 (fg) = \vec{\nabla} \cdot \vec{\nabla} (fg) = g \vec{\nabla}^2 f + f \vec{\nabla}^2 g + 2(\vec{\nabla} f) \cdot (\vec{\nabla} g)$ }

Let $\xi = t - \sqrt{\mu\epsilon} R = t - R/u$, $\rho(\xi) = \rho(t - R/u)$

$$\vec{\nabla}^2 \rho(\xi) = \frac{1}{u^2} \frac{d^2 \rho}{d\xi^2} - \frac{2}{uR} \frac{d\rho}{d\xi}, \quad \vec{\nabla}^2 (\frac{1}{R}) = -4\pi \delta(R)$$

$$(\bar{\nabla} \rho) \cdot (\bar{\nabla} \frac{1}{R}) = \frac{\partial \rho(\psi)}{\partial R} \left(-\frac{1}{R^2}\right) = -\frac{1}{u R^2} \frac{d\rho}{d\psi}$$

Substituting back, $\bar{\nabla}^2 \left(\frac{\rho}{R}\right) = \frac{1}{u^2 R} \frac{d^2 \rho}{d\psi^2} - 4\pi \rho \delta(R)$.

$$\bar{\nabla}^2 V = \frac{1}{4\pi\epsilon} \bar{\nabla}^2 \int_{V'} \frac{\rho}{R} dv' = \frac{1}{4\pi\epsilon} \int_{V'} \left[\frac{1}{u^2 R} \frac{d^2 \rho}{d\psi^2} - 4\pi \rho \delta(R) \right] dv'$$

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{4\pi\epsilon} \int_{V'} \frac{1}{R} \frac{d^2 \rho}{d\psi^2} dv'$$

$$\begin{aligned} \therefore \bar{\nabla}^2 V - \frac{1}{u^2} \frac{\partial^2 V}{\partial t^2} &= \frac{1}{4\pi\epsilon} \int \left[\frac{1}{u^2 R} \frac{d^2 \rho}{d\psi^2} - 4\pi \rho \delta(R) - \frac{1}{u^2 R} \frac{d^2 \rho}{d\psi^2} \right] dv' \\ &= -\frac{\rho(R)}{\epsilon} \quad \text{Q.E.D.} \end{aligned}$$

P. 7-17 $\bar{\nabla} \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$ ① $\bar{\nabla} \times \bar{H} = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t}$ ②

$\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon}$ ③ $\bar{\nabla} \cdot \bar{H} = 0$ ④

$\bar{\nabla} \times$ ①: $\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -\mu \frac{\partial}{\partial t} (\bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t})$.

$\bar{\nabla} (\bar{\nabla} \cdot \bar{E}) - \bar{\nabla}^2 \bar{E} = -\mu \frac{\partial \bar{J}}{\partial t} + \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$.

Wave equation for \bar{E} : $\bar{\nabla}^2 \bar{E} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = \mu \frac{\partial \bar{J}}{\partial t} + \frac{1}{\epsilon} \bar{\nabla} \rho$.

$\bar{\nabla} \times$ ②: $\bar{\nabla} \times \bar{\nabla} \times \bar{H} = \bar{\nabla} \times \bar{J} + \epsilon \frac{\partial}{\partial t} (\bar{\nabla} \times \bar{E})$.

$\bar{\nabla} (\bar{\nabla} \cdot \bar{H}) - \bar{\nabla}^2 \bar{H} = \bar{\nabla} \times \bar{J} - \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2}$.

Wave equation for \bar{H} : $\bar{\nabla}^2 \bar{H} - \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = -\bar{\nabla} \times \bar{J}$.

For sinusoidal time dependence: $\frac{\partial}{\partial t} \rightarrow j\omega$, $\frac{\partial^2}{\partial t^2} \rightarrow -(\omega)^2$.

Helmholtz's equations: $\bar{\nabla}^2 \bar{E} + \omega^2 \mu \epsilon \bar{E} = j\omega \mu \bar{J} + \frac{1}{\epsilon} \bar{\nabla} \rho$

(for phasors) $\bar{\nabla}^2 \bar{H} + \omega^2 \mu \epsilon \bar{H} = -\bar{\nabla} \times \bar{J}$.

P. 7-18 $\bar{E} = \bar{a}_y 0.1 \sin(10\pi x) \cos(6\pi 10^9 t - \beta z)$ (V/m)

Use phasors:

$$\bar{H} = -\frac{1}{j\omega\mu_0} \bar{\nabla} \times \bar{E} = -\frac{1}{j\omega\mu_0} \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \frac{j}{\omega\mu_0} [\bar{a}_z j 0.1 \beta \sin(10\pi x) + \bar{a}_x 0.1 (10\pi) \cos(10\pi x)] e^{-j\beta z} \quad \text{①}$$

$$\bar{E} = \frac{1}{j\omega\epsilon_0} \bar{\nabla} \times \bar{H} = \bar{a}_y \frac{0.1}{\omega^2 \mu_0 \epsilon_0} [(10\pi)^2 + \beta^2] \sin(10\pi x) e^{-j\beta z} \quad \text{②}$$

Phasor form for given \vec{E} :

$$\vec{E} = \bar{a}_y 0.1 \sin(10\pi x) e^{j\beta z} \quad (3)$$

Equating ② and ③: $(10\pi)^2 + \beta^2 = \omega^2 \mu_0 \epsilon_0 = 400\pi^2$

$$\therefore \beta = \sqrt{300} \pi = 10\sqrt{3} \pi = 54.4 \text{ (rad/m)}$$

From ①, $\vec{H}(x, z; t) = \text{Re}(H e^{j\omega t})$

$$= -\bar{a}_x 2.30 \times 10^{-4} \sin(10\pi x) \cos(6\pi 10^9 t - 54.4z)$$

$$- \bar{a}_z 1.33 \times 10^{-4} \cos(10\pi x) \sin(6\pi 10^9 t - 54.4z) \quad (\text{A/m})$$

P.7-19 $\vec{H}(x, z; t) = \bar{a}_y 2 \cos(15\pi x) \sin(6\pi 10^9 t - \beta z) \quad (\text{A/m})$

Phasor: $\vec{H} = \bar{a}_y 2 \cos(15\pi x) e^{-j\beta z}$

$$\beta^2 + (15\pi)^2 = \omega^2 \mu_0 \epsilon_0 = (6\pi 10^9)^2 \frac{1}{(3 \times 10^8)^2}$$

$$\beta^2 = 400\pi^2 - 225\pi^2 = 175\pi^2$$

$$\beta = 13.2\pi = 41.6 \text{ (rad/m)}$$

$$\vec{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \vec{H} = \frac{1}{j\omega\epsilon_0} (-\bar{a}_x \frac{\partial H_y}{\partial z} + \bar{a}_z \frac{\partial H_x}{\partial x})$$

$$= -j6 [-\bar{a}_x j2\beta \cos(15\pi x) - \bar{a}_z 30\pi \sin(15\pi x)] e^{-j\beta z}$$

$$= [-\bar{a}_x 158\pi \cos(15\pi x) + \bar{a}_z j180\pi \sin(15\pi x)] e^{-j\beta z}$$

$$\vec{E}(x, z; t) = \text{Im}(\vec{E} e^{j\omega t})$$

$$= \bar{a}_x 496 \cos(15\pi x) \sin(6\pi 10^9 t - 41.6z)$$

$$+ \bar{a}_z 565 \sin(15\pi x) \cos(6\pi 10^9 t - 41.6z) \quad (\text{V/m})$$

P.7-20 $\vec{E} = \bar{a}_\phi \frac{E_0}{R} \sin\theta \cos(\omega t - kr) \quad (1)$

$$\nabla \times \vec{E} = \bar{a}_\phi \frac{1}{R} \frac{\partial}{\partial R} (R E_\phi) = \bar{a}_\phi \frac{E_0 k}{R} \sin\theta \sin(\omega t - kr)$$

$$= -\mu_0 \frac{\partial \vec{H}}{\partial t} \rightarrow \vec{H} = \bar{a}_\phi \int -\frac{E_0 k}{\mu_0 R} \sin\theta \sin(\omega t - kr) dt$$

$$= \bar{a}_\phi \frac{E_0 k}{\omega \mu_0 R} \sin\theta \cos(\omega t - kr) \quad (2)$$

Also, $\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow \frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon_0} \nabla \times \vec{H} = -\bar{a}_\phi \frac{1}{R} \frac{\partial}{\partial R} (R H_\phi)$

$$\vec{E} = \bar{a}_\phi \int -\frac{E_0 k^2}{\omega \mu_0 \epsilon_0 R} \sin\theta \sin(\omega t - kr) dt$$

Comparing ① and ②,

$$k = \omega \sqrt{\mu_0 \epsilon_0}$$

$$= \bar{a}_\phi \frac{E_0 k^2}{\omega^2 \mu_0 \epsilon_0 R} \sin\theta \sin(\omega t - kr) \quad (3)$$

From ②, $\vec{H} = \bar{a}_\phi \frac{E_0}{R} \sqrt{\frac{\epsilon_0}{\mu_0}} \sin\theta \cos \omega(t - \sqrt{\mu_0 \epsilon_0} r)$

P.7-21 Maxwell's curl equations: $\nabla \times \bar{E} = -j\omega\mu\bar{H}$ ①

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E}$$
 ②

From $\nabla \cdot \bar{E} = 0$, define \bar{A}_e such that $\bar{E} = -\nabla \times \bar{A}_e$ ③

From ①, $\bar{H} = \frac{j}{\omega\mu} \nabla \times \bar{E} = \frac{j}{\omega\mu} \nabla \times \nabla \times \bar{A}_e$

$$= \frac{j}{\omega\mu} [\nabla(\nabla \cdot \bar{A}_e) - \nabla^2 \bar{A}_e].$$
 ④

From ②, $\nabla \times (\bar{H} - j\omega\epsilon\bar{A}_e) = 0$, let $\bar{H} - j\omega\epsilon\bar{A}_e = -\nabla V_m$. ⑤

Subtracting ⑤ from ④: $\omega\epsilon\bar{A}_e = \frac{j}{\omega\mu} [\nabla(\nabla \cdot \bar{A}_e) - \nabla^2 \bar{A}_e] - j\nabla V_m$. ⑥

Choose $\nabla \cdot \bar{A}_e = j\omega\mu V_m$.

a) Eq. ⑤ becomes $\bar{H} = j\omega\epsilon\bar{A}_e + \frac{j}{\omega\mu} \nabla(\nabla \cdot \bar{A}_e)$.

b) Eq. ⑥ becomes $\nabla^2 \bar{A}_e + \omega^2\mu\epsilon\bar{A}_e = 0$, a homogeneous Helmholtz eq.

P.7-22 $\bar{H} = j\omega\epsilon_0 \nabla \times \bar{\pi}_e$ ①

$$\nabla \times \bar{E} = -j\omega\mu_0\bar{H} = \omega^2\mu_0\epsilon_0 \nabla \times \bar{\pi}_e$$

$$\nabla \times (\bar{E} - k_0^2 \bar{\pi}_e) = 0 \longrightarrow \text{Let } \bar{E} - k_0^2 \bar{\pi}_e = \nabla V_e$$
 ②

$$\nabla \times \bar{H} = j\omega\bar{D} = j\omega(\epsilon_0\bar{E} + \bar{P}) = j\omega\epsilon_0(\bar{E} + \frac{\bar{P}}{\epsilon_0}).$$
 ③

Substituting ① and ③ in ②:

$$j\omega\epsilon_0 \nabla \times \nabla \times \bar{\pi}_e = j\omega\epsilon_0 (k_0^2 \bar{\pi}_e + \nabla V_e + \frac{\bar{P}}{\epsilon_0})$$

$$= j\omega\epsilon_0 (\nabla \nabla \cdot \bar{\pi}_e - \nabla^2 \bar{\pi}_e).$$
 ④

Choose $\nabla \cdot \bar{\pi}_e = V_e$. Eq. ④ becomes

b) $\nabla^2 \bar{\pi}_e + k_0^2 \bar{\pi}_e = -\frac{\bar{P}}{\epsilon_0}$; (7-95)

a) Eq. ② becomes

$$\bar{E} = k_0^2 \bar{\pi}_e + \nabla \nabla \cdot \bar{\pi}_e$$

$$= k_0^2 \bar{\pi}_e + (\nabla^2 \bar{\pi}_e + \nabla \times \nabla \times \bar{\pi}_e).$$
 ⑤

Combination of Eqs. (7-95) and ⑤ gives

$$\bar{E} = \nabla \times \nabla \times \bar{\pi}_e - \frac{\bar{P}}{\epsilon_0}.$$

P.7-23 a) $\left| \frac{\text{Displacement current}}{\text{Conduction current}} \right| = \frac{\omega\epsilon}{\sigma} = \frac{(2\pi 100 \times 10^3) \times \frac{1}{36\pi} \times 10^{-9}}{5.70 \times 10^7} = 9.75 \times 10^{-3}$

b) $\nabla \times \bar{H} = \sigma\bar{E}$, $\nabla \times \bar{E} = -j\omega\mu\bar{H}$

$$\nabla \times \nabla \times \bar{H} = \nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H} = \sigma \nabla \times \bar{E}.$$

But $\nabla \cdot \bar{H} = 0$.

$\therefore \nabla^2 \bar{H} + \sigma \nabla \times \bar{E} = 0$,

or, $\nabla^2 \bar{H} - j\omega\mu\sigma \bar{H} = 0$.

Chapter 8

P.8-2 Harmonic time dependence: $e^{j\omega t}$; $\frac{\partial}{\partial t} \rightarrow j\omega$

8-4

Let phasors $\bar{E} = \bar{E}_0 e^{j\bar{k} \cdot \bar{r}}$

$\bar{H} = \bar{H}_0 e^{j\bar{k} \cdot \bar{r}}$,

where \bar{E}_0 and \bar{H}_0 are constant vectors.

Maxwell's equations: $\nabla \times \bar{E} = \nabla (e^{j\bar{k} \cdot \bar{r}}) \times \bar{E}_0 = -j\omega\mu \bar{H}$

$\nabla \times \bar{H} = \nabla (e^{j\bar{k} \cdot \bar{r}}) \times \bar{H}_0 = j\omega\epsilon \bar{E}$

$\nabla \cdot \bar{E} = \nabla (e^{j\bar{k} \cdot \bar{r}}) \cdot \bar{E}_0 = 0$

$\nabla \cdot \bar{H} = \nabla (e^{j\bar{k} \cdot \bar{r}}) \cdot \bar{H}_0 = 0$.

But $\nabla (e^{j\bar{k} \cdot \bar{r}}) = e^{j\bar{k} \cdot \bar{r}} \nabla (-j\bar{k} \cdot \bar{r}) = e^{j\bar{k} \cdot \bar{r}} [-j \nabla (k_x x + k_y y + k_z z)]$
 $= -j(\bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z) e^{j\bar{k} \cdot \bar{r}} = -j\bar{k} e^{j\bar{k} \cdot \bar{r}}$

\therefore Maxwell's equations become: $\bar{k} \times \bar{E} = \omega\mu \bar{H}$, $\bar{k} \times \bar{H} = -\omega\epsilon \bar{E}$
 $\bar{k} \cdot \bar{E} = 0$, $\bar{k} \cdot \bar{H} = 0$.

8-5

P.8-3 $\bar{H} = \bar{a}_z 4 \times 10^{-6} \cos(10^7 \pi t - k_0 y + \frac{\pi}{4})$ (A/m).

a) $k_0 = \omega \sqrt{\mu_0 \epsilon_0} = \frac{10^7 \pi}{3 \times 10^8} = \frac{\pi}{30} = 0.105$ (rad/m).

At $t = 3 \times 10^{-3}$ (s), we require

$10^7 \pi (3 \times 10^{-3}) - \frac{\pi}{30} y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}$, $n=0, 1, 2, \dots$

$y = \pm 30n - 7.5$ (m).

But $\lambda = \frac{2\pi}{k_0} = 60$ (m), $\therefore y = 22.5 \pm n\lambda/2$ (m).

b) $\bar{E} = \eta_0 \bar{H} \times \bar{a}_y = -\bar{a}_z 1.508 \times 10^{-3} \cos(10^7 \pi t - \frac{\pi}{30} y + \frac{\pi}{4})$ (V/m).

8-7

P.8-4 Let $\alpha = \omega t - kz$, $\bar{E} = \bar{a}_x E_{10} \sin \alpha + \bar{a}_y E_{20} \sin(\alpha + \psi)$
 $= \bar{a}_x E_x + \bar{a}_y E_y$.

$\frac{E_x}{E_{10}} = \sin \alpha$, $\frac{E_x}{E_{20}} = \sin(\alpha + \psi) = \sin \alpha \cos \psi + \cos \alpha \sin \psi$
 $= \frac{E_x}{E_{20}} \cos \psi + \sqrt{1 - (\frac{E_x}{E_{20}})^2} \sin \psi$.

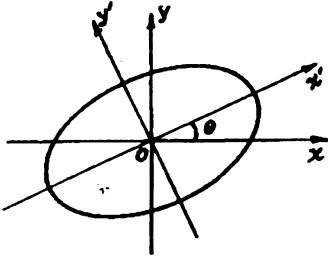
8-7. cont

$$\left(\frac{E_y}{E_{10}} - \frac{E_x}{E_0} \cos \psi\right)^2 = \left(1 - \frac{E_x}{E_{10}}\right) \sin^2 \psi$$

$$\left(\frac{E_y}{E_{20} \sin \psi}\right)^2 + \left(\frac{E_x}{E_{10} \sin \psi}\right)^2 - 2 \frac{E_x E_y}{E_0 E_{20}} \frac{\cos \psi}{\sin^2 \psi} = 1, \quad (1)$$

which is the equation of an ellipse.

In order to find the parameters of the ellipse, rotate the coordinate axes x - y counterclockwise by an angle θ to x' - y' . Assume the equation of



the ellipse in terms of the new coordinates to be

$$\left(\frac{E'_x}{a}\right)^2 + \left(\frac{E'_y}{b}\right)^2 = 1, \quad (2)$$

where

$$E'_x = E_x \cos \theta + E_y \sin \theta \quad (3)$$

$$E'_y = -E_x \sin \theta + E_y \cos \theta. \quad (4)$$

Substituting (3) and (4) in (2) and rearranging:

$$E_x^2 \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}\right) + E_y^2 \left(\frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2}\right) - 2 E_x E_y \sin \theta \cos \theta \left(\frac{1}{b^2} - \frac{1}{a^2}\right) = 1. \quad (5)$$

Comparing (1) and (5), we obtain

$$\left\{ \begin{array}{l} \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{E_{10}^2 \sin^2 \psi} \\ \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} = \frac{1}{E_{20}^2 \sin^2 \psi} \end{array} \right. \quad (6)$$

$$\left\{ \begin{array}{l} \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{E_{10}^2 \sin^2 \psi} \\ \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} = \frac{1}{E_{20}^2 \sin^2 \psi} \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{E_{10}^2 \sin^2 \psi} \\ \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2} = \frac{1}{E_{20}^2 \sin^2 \psi} \\ \sin \theta \cos \theta \left(\frac{1}{b^2} - \frac{1}{a^2}\right) = \frac{\cos \psi}{E_0 E_{20} \sin^2 \psi} \end{array} \right. \quad (8)$$

Eqs. (6), (7) and (8) can be solved for three unknowns:

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2 E_{10} E_{20} \cos \psi}{E_{10}^2 - E_{20}^2} \right)$$

$$a = \sqrt{\frac{2}{\frac{1}{E_{10}^2} (1 + \sec 2\theta) + \frac{1}{E_{20}^2} (1 - \sec 2\theta)}} \sin \psi$$

$$b = \sqrt{\frac{2}{\frac{1}{E_{10}^2} (1 - \sec 2\theta) + \frac{1}{E_{20}^2} (1 + \sec 2\theta)}} \sin \psi.$$

In particular, if $E_{10} = E_{20}$: $\theta = 45^\circ$.

$$= E_0, \quad a = \sqrt{2} E_0 \cos \frac{\psi}{2}, \quad b = \sqrt{2} E_0 \sin \frac{\psi}{2}.$$

P.8-5 Let an elliptically polarized plane wave be represented by the phasor (with propagation factor e^{-jkz} omitted):

$$a) \quad \bar{E} = \bar{a}_x E_1 \pm \bar{a}_y E_2 e^{j\alpha}$$

where E_1 , E_2 , and α are arbitrary constants.

Right-hand circularly polarized wave: $\bar{E}_{rc} = E_{rc} (\bar{a}_x - j\bar{a}_y)$.

Left-hand circularly polarized wave: $\bar{E}_{lc} = E_{lc} (\bar{a}_x + j\bar{a}_y)$.

$$\text{If } E_{rc} = \frac{1}{2}(E_1 \pm jE_2 e^{j\alpha})$$

$$E_{lc} = \frac{1}{2}(E_1 \mp jE_2 e^{j\alpha}),$$

$$\text{then } \bar{E} = \bar{E}_{rc} + \bar{E}_{lc}.$$

b) Let a right-hand circularly polarized wave be

$$\bar{E}_{rc} = E(\bar{a}_x - j\bar{a}_y)$$

$$= E(\bar{a}_x \frac{1}{2} - \bar{a}_y j) + E(\bar{a}_x \frac{1}{2} + \bar{a}_y j)$$

$$= \bar{E}_{e+} + \bar{E}_{e-},$$

where E_{e+} and E_{e-} are right-hand and left-hand elliptically polarized waves respectively.

Similarly, a left-hand circularly polarized wave can be written as

$$\bar{E}_{lc} = E(\bar{a}_x + j\bar{a}_y)$$

$$= E(\bar{a}_x \frac{1}{2} + \bar{a}_y j) + E(\bar{a}_x \frac{1}{2} - \bar{a}_y j)$$

$$= \bar{E}'_{e-} + \bar{E}'_{e+}.$$

P.8-6 For conducting media,

$$k_c^2 = \omega^2 \mu \epsilon_c = \omega^2 \mu \epsilon (1 - j \frac{\sigma}{\omega \epsilon})$$

$$k_c = \beta - j\alpha, \quad k_c^2 = \beta^2 - \alpha^2 - 2j\alpha\beta.$$

$$\therefore \beta^2 - \alpha^2 = \text{Re}(k_c^2) = \omega^2 \mu \epsilon \quad (1)$$

$$\beta^2 + \alpha^2 = |k_c^2| = \omega^2 \mu \epsilon \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}. \quad (2)$$

From (1) and (2) we obtain

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1 \right]^{1/2}}, \quad \beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1 \right]^{1/2}}.$$

8-7 All are good conductors, $\left(\frac{\sigma}{\omega\epsilon}\right)^2 \gg 1$.

$$\alpha = \sqrt{\pi f \mu \sigma}, \quad \delta = \frac{1}{\alpha}, \quad \eta_c = (1+j) \frac{\alpha}{\sigma}$$

a) $f = 60 \text{ (Hz)}$

	$\eta_c \text{ (}\Omega\text{)}$	$\alpha \text{ (Np/m)}$	$\alpha \text{ (dB/m)}$	$\delta \text{ (m)}$
Copper	$2.02(1+j) \times 10^{-3}$	0.117×10^3	1.02×10^3	8.53×10^{-3}
Silver	$2.08(1+j) \times 10^{-3}$	0.121×10^3	1.05×10^3	8.29×10^{-3}
Brass	$3.86(1+j) \times 10^{-3}$	0.061×10^3	0.53×10^3	16.3×10^{-3}

b) $f = 1 \text{ (MHz)}$

	$\eta_c \text{ (}\Omega\text{)}$	$\alpha \text{ (Np/m)}$	$\alpha \text{ (dB/m)}$	$\delta \text{ (m)}$
Copper	$2.61(1+j) \times 10^{-3}$	1.51×10^4	1.31×10^5	6.61×10^{-5}
Silver	$2.57(1+j) \times 10^{-3}$	1.58×10^4	1.35×10^5	6.32×10^{-5}
Brass	$4.98(1+j) \times 10^{-3}$	0.79×10^4	0.69×10^5	12.6×10^{-5}

c) $f = 1 \text{ (GHz)}$

	$\eta_c \text{ (}\Omega\text{)}$	$\alpha \text{ (Np/m)}$	$\alpha \text{ (dB/m)}$	$\delta \text{ (m)}$
Copper	$8.25(1+j) \times 10^{-3}$	4.79×10^5	4.16×10^6	2.09×10^{-6}
Silver	$8.01(1+j) \times 10^{-3}$	4.93×10^5	4.28×10^6	2.03×10^{-6}
Brass	$15.8(1+j) \times 10^{-3}$	2.51×10^5	2.18×10^6	3.99×10^{-6}

8-8
8-11

a) $f = 3 \times 10^9 \text{ (Hz)}, \epsilon_r = 2.5, \tan \delta_c = \frac{\sigma}{\omega\epsilon} = 10^{-2}$.

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]^{1/2} \cong \omega \sqrt{\frac{\mu\epsilon}{2}} \frac{1}{\sqrt{2}} \left(\frac{\sigma}{\omega\epsilon}\right)^2$$

$$= 0.497 \text{ (Np/m)}.$$

$$e^{-\alpha x} = \frac{1}{2} \rightarrow x = \frac{1}{\alpha} \ln 2 = 1.395 \text{ (m)}.$$

b) $\eta_c = \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu}{\epsilon_0}} \left(1 + j \frac{\sigma}{2\omega\epsilon}\right) = 238(1 + j0.005) \text{ (}\Omega\text{)}$

$$\beta = \omega \sqrt{\mu\epsilon} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega\epsilon}\right)^2\right] = 31.6\pi \text{ (rad/m)}$$

$$\lambda = \frac{2\pi}{\beta} = 0.063 \text{ (m)}$$

$$u_p = \frac{\omega}{\beta} = 1.8973 \times 10^8 \text{ (m/s)}$$

$$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}} \left[1 + \frac{1}{8} \left(\frac{\sigma}{\omega\epsilon}\right)^2\right] = 1.8975 \times 10^8 \text{ (m/s)}.$$

p-8 cont.

c) At $x=0$, $\bar{E} = \bar{a}_y 50 e^{j\pi/3}$

$$\bar{H} = \frac{1}{\eta_c} \bar{a}_x \times \bar{E} = \bar{a}_x 0.210 e^{j(\frac{\pi}{3} - 0.0016\pi)}$$

$$\therefore \bar{H} = \bar{a}_x 0.210 e^{-0.0016\pi} \sin(6\pi \times 10^8 t - 31.6\pi x + \frac{\pi}{3} - 0.0016\pi) \quad (\text{A/m})$$

P.8-9 a) $\frac{\sigma}{\omega \epsilon} = 0.18$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1 \right]^{1/2} = 84 \text{ (Np/m)}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1 \right]^{1/2} = 300\pi \text{ (rad/m)}$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r \left[1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2\right]^{1/2}}} e^{j \tan^{-1}(\sigma/\omega \epsilon)} = 41.8 e^{j0.0253\pi} \text{ (\Omega)}$$

$$u_p = \frac{\omega}{\beta} = 33.3 \times 10^6 \text{ (m/s)}$$

$$\lambda = \frac{2\pi}{\beta} = 6.67 \times 10^{-3} \text{ (m)}$$

$$\delta = \frac{1}{\alpha} = 11.9 \times 10^{-3} \text{ (m)}$$

b) $e^{-\alpha y} = \frac{1}{10}$, $y = \frac{1}{\alpha} \ln 10 = 27.4 \times 10^{-3} \text{ (m)}$

c) At $y = 0.5 \text{ (m)}$,

$$\bar{H}(y,t) = \bar{a}_x 0.1 e^{-84 \times 0.5} \sin(10^{10} \pi t - 300\pi \times 0.5 - \frac{\pi}{3})$$

$$= \bar{a}_x 5.75 \times 10^{-20} \sin(10^{10} \pi t - \frac{\pi}{3}) \text{ (A/m)}$$

$$\bar{E}(y,t) = \text{Im}[\eta_c \bar{H}(y) \times \bar{a}_y] e^{j\omega t} = \bar{a}_y 2.41 \times 10^{-20} \sin(10^{10} \pi t - \frac{\pi}{3} + 0.029\pi) \text{ (V/m)}$$

P.8-10 a) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$$\sigma = \frac{1}{\pi f \mu \delta^2} = 0.99 \times 10^5 \text{ (S/m)}$$

b) At $f = 10^9 \text{ (Hz)}$, $\alpha = \sqrt{\pi f \mu \sigma} = 1.98 \times 10^4 \text{ (Np/m)}$

$$20 \log_{10} e^{-\alpha z} = -30 \text{ (dB)}$$

$$z = \frac{1.5}{\alpha \log_{10} e} = 1.75 \times 10^{-4} \text{ (m)}$$

P.8-11 a) From Eq. (8-52), $u_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta u_p) = u_p + \beta \frac{du_p}{d\beta}$

b) $\lambda = \frac{2\pi}{\beta}$, $\frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta}$

$$u_g = u_p + \beta \left(\frac{du_p}{d\lambda} \frac{d\lambda}{d\beta} \right) = u_p - \lambda \frac{du_p}{d\lambda}$$

8-12 $\mathcal{P}_{av} = \frac{|E|^2}{2\eta_0} = 10^{-3} \text{ (W/cm}^2\text{)}$

a) $|E| = \sqrt{0.02 \eta_0} = 2.75 \text{ (V/cm)} = 275 \text{ (V/m)}$
 $|H| = \frac{1}{\eta_0} |E| = 7.28 \times 10^{-3} \text{ (A/cm)} = 0.728 \text{ (A/m)}$

b) $\mathcal{P}_{av} = \frac{|E|^2}{2\eta_0} = 1.3 \times 10^3 \text{ (W/cm}^2\text{)}$
 $|E| = 990 \text{ (V/cm)} = 9.90 \times 10^4 \text{ (V/m)}$
 $|H| = 2.63 \text{ (A/cm)} = 263 \text{ (A/m)}$

8-13 Assume that a circularly polarized plane wave be represented by

$$\bar{E}(z, t) = \bar{a}_x E_0 \cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi)$$

$$\bar{H}(z, t) = \bar{a}_y \frac{E_0}{\eta} \cos(\omega t - kz + \phi) - \bar{a}_x \frac{E_0}{\eta} \sin(\omega t - kz + \phi)$$

The Poynting vector is

$$\bar{P} = \bar{E} \times \bar{H} = \bar{a}_z \frac{E_0^2}{\eta} [\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)]$$

$$= \bar{a}_z \frac{E_0^2}{\eta}, \text{ a constant independent of } t \text{ and } z.$$

8-14 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$

$$\bar{H} = \frac{1}{\eta} \bar{a}_r \times \bar{E} = \frac{1}{\eta} (\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi)$$

$$\bar{P}_{av} = \frac{1}{2} \Re(\bar{E} \times \bar{H}^*) = \bar{a}_z \frac{1}{2\eta} (|E_\theta|^2 + |E_\phi|^2) \text{ (W/m}^2\text{)}$$

8-15 From Gauss's law, $\bar{E} = \bar{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$V_0 = -\int_b^a \bar{E} \cdot d\bar{r} = \frac{\rho}{2\pi\epsilon} \ln\left(\frac{b}{a}\right)$$

$$\therefore \bar{E} = \bar{a}_r \frac{V_0}{r \ln(b/a)}$$

From Ampère's circuital law, $\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}$

$$\text{Poynting vector } \bar{P} = \bar{E} \times \bar{H} = \bar{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}$$

Power transmitted of cross-sectional area:

$$P = \int_S \bar{P} \cdot d\bar{s} = \frac{V_0 I}{2\pi \ln(b/a)} \int_0^{2\pi} \int_a^b \left(\frac{1}{r^2}\right) r dr d\phi = V_0 I$$

P.8-16 Given $\vec{E}_i = E_0 (\bar{a}_x - j\bar{a}_y) e^{j\beta z}$.

- a) Assume the reflected electric field intensity to be

$$\vec{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry}) e^{j\beta z}$$

Boundary condition at $z=0$:

$$[\vec{E}_i(z) + \vec{E}_r(z)]_{z=0} = 0$$

$\therefore \vec{E}_r(z) = E_0 (-\bar{a}_x + j\bar{a}_y) e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

- b) $\bar{a}_{n2} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \longrightarrow -\bar{a}_x \times [\vec{H}_1(0) + \vec{H}_r(0)] = \vec{J}_s$

$$\vec{H}_1(0) = \frac{1}{\eta_0} \bar{a}_z \times \vec{E}_i(0) = \frac{E_0}{\eta_0} (j\bar{a}_x + \bar{a}_y) \quad \vec{H}_2 = 0 \text{ in perfect conductor}$$

$$\vec{H}_r(0) = \frac{1}{\eta_0} (-\bar{a}_z) \times \vec{E}_r(0) = \frac{E_0}{\eta_0} (j\bar{a}_x + \bar{a}_y)$$

$$\vec{H}_i(0) = \vec{H}_1(0) + \vec{H}_r(0) = \frac{2E_0}{\eta_0} (j\bar{a}_x + \bar{a}_y)$$

$$\therefore \vec{J}_s = -\bar{a}_x \times \vec{H}_i(0) = \frac{2E_0}{\eta_0} (\bar{a}_x - j\bar{a}_y)$$

- c) $\vec{E}_i(z,t) = \text{Re} [E_i(z) + E_r(z)] e^{j\omega t}$
 $= \text{Re} E_0 [(\bar{a}_x - j\bar{a}_y) e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y) e^{j\beta z}] e^{j\omega t}$
 $= \text{Re} E_0 [-2j(\bar{a}_x - j\bar{a}_y) \sin\beta z] e^{j\omega t}$
 $= 2E_0 \sin\beta z (\bar{a}_x \sin\omega t - \bar{a}_y \cos\omega t)$

P.8-17 Given $\vec{E}_i(x,z) = \bar{a}_y 10 e^{-j(6x+8z)}$ (V/m).

8-22 a) $k_x = 6, k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10} = 0.628 \text{ (m)}$$

$$f = \frac{c}{\lambda} = 4.78 \times 10^8 \text{ (Hz)}; \quad \omega = 2\pi f = kc = 3 \times 10^9 \text{ (rad/s)}$$

b) $\vec{E}_i(x,z;t) = \bar{a}_y 10 \cos(3 \times 10^9 t - 6x - 8z)$ (V/m)

$$\vec{H}_i(x,z) = \frac{1}{\eta_0} \bar{a}_{ni} \times \vec{E}_i \quad \bar{a}_{ni} = \frac{\vec{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8$$

$$= \frac{1}{120\pi} (\bar{a}_x 0.6 + \bar{a}_z 0.8) \times \bar{a}_y 10 e^{j(6x+8z)}$$

$$= (-\bar{a}_x \frac{1}{15\pi} + \bar{a}_z \frac{1}{20\pi}) e^{j(6x+8z)}$$

$$\vec{H}_i(x,z;t) = (-\bar{a}_x \frac{1}{15\pi} + \bar{a}_z \frac{1}{20\pi}) \cos(3 \times 10^9 t - 6x - 8z) \text{ (A/m)}$$

c) $\cos\theta_i = \bar{a}_{ni} \cdot \bar{a}_z = \left(\frac{\vec{k}}{k}\right) \cdot \bar{a}_z = (\bar{a}_x 0.6 + \bar{a}_z 0.8) \cdot \bar{a}_z = 0.8$

Cont. 8.22

$$\theta_i = \cos^{-1} 0.8 = 36.9^\circ$$

$$d) \bar{E}_i(x, 0) + \bar{E}_r(x, 0) = 0 \longrightarrow \bar{E}_r(x, z) = -\bar{a}_y 10 e^{-j(6x-8z)} \quad (\text{V/m})$$

$$\begin{aligned} \bar{H}_r(x, z) &= \frac{1}{\eta_0} \bar{a}_{nr} \times \bar{E}_r(x, z) & \bar{a}_{nr} &= \bar{a}_z 0.6 + \bar{a}_x 0.8 \\ &= -(\bar{a}_x \frac{1}{15\pi} - \bar{a}_z \frac{1}{30\pi}) e^{-j(6x-8z)} \quad (\text{A/m}) \end{aligned}$$

$$e) \bar{E}_r(x, z) = \bar{E}_i(x, z) + \bar{E}_r(x, z) = \bar{a}_y 10 (e^{-j8z} - e^{j8z}) e^{-j6x} \\ = -\bar{a}_y j 20 e^{-j6x} \sin 8z \quad (\text{V/m})$$

$$\begin{aligned} \bar{H}_i(x, z) &= \bar{H}_i(x, z) + \bar{H}_r(x, z) \\ &= -(\bar{a}_x \frac{2}{15\pi} \cos 8z - \bar{a}_z \frac{j}{10\pi} \sin 8z) e^{-j6x} \quad (\text{A/m}) \end{aligned}$$

8-18 Given $\bar{E}_i(y, z) = 5(\bar{a}_y + \bar{a}_z \sqrt{3}) e^{j6(\sqrt{3}y - z)}$ (V/m):

$$a) k_y = -6\sqrt{3}, \quad k_z = 6 \longrightarrow k = \sqrt{k_y^2 + k_z^2} = 12 \text{ (rad/m)}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{12} = \frac{\pi}{6} = 0.524 \text{ (m)}$$

$$f = \frac{c}{\lambda} = 5.73 \times 10^8 \text{ (Hz)}; \quad \omega = 2\pi f = kc = 3.6 \times 10^9 \text{ (rad/s)}$$

$$b) \bar{E}_i(y, z; t) = 5(\bar{a}_y + \bar{a}_z \sqrt{3}) \cos(3.6 \times 10^9 t + 6\sqrt{3}y - 6z) \quad (\text{V/m})$$

$$\begin{aligned} \bar{H}_i(y, z) &= \frac{1}{\eta_0} \bar{a}_{ni} \times \bar{E}_i = \frac{1}{120\pi} (-\bar{a}_y \frac{\sqrt{3}}{2} + \bar{a}_z \frac{1}{2}) \times 5(\bar{a}_y + \bar{a}_z \sqrt{3}) e^{j6(\sqrt{3}y - z)} \\ &= \bar{a}_x (-\frac{1}{12\pi}) e^{j6(\sqrt{3}y - z)} \end{aligned}$$

$$\bar{H}_i(y, z; t) = \bar{a}_x (-\frac{1}{12\pi}) \cos(3.6 \times 10^9 t + 6\sqrt{3}y - 6z) \quad (\text{A/m})$$

$$c) \cos \theta_i = \bar{a}_{ni} \cdot \bar{a}_z = \frac{1}{2} \longrightarrow \theta_i = \cos^{-1}(\frac{1}{2}) = 60^\circ$$

$$d) \bar{a}_{nr} \times \bar{E}_r(y, z) = 0 \text{ and } E_{iy}(y, 0) + E_{ry}(y, 0) = 0 \text{ lead to:}$$

$$\bar{E}_r(y, z) = 5(-\bar{a}_y + \bar{a}_z \sqrt{3}) e^{j6(\sqrt{3}y + z)} \quad (\text{V/m})$$

$$\begin{aligned} \bar{H}_r(y, z) &= \frac{1}{\eta_0} \bar{a}_{nr} \times \bar{E}_r(y, z) \\ &= \frac{1}{120\pi} (-\bar{a}_y \frac{\sqrt{3}}{2} - \bar{a}_z \frac{1}{2}) \times 5(-\bar{a}_y + \bar{a}_z \sqrt{3}) e^{j6(\sqrt{3}y + z)} \\ &= \bar{a}_x (-\frac{1}{12\pi}) e^{j6(\sqrt{3}y + z)} \quad (\text{A/m}) \end{aligned}$$

$$e) \bar{E}_i(y, z) = \bar{E}_i(y, z) + \bar{E}_r(y, z) \\ = (\bar{a}_y (-10j) \sin 6z + \bar{a}_z 10\sqrt{3} \cos 6z) e^{j6\sqrt{3}y}$$

$$\begin{aligned} \bar{H}_i(y, z) &= \bar{H}_i(y, z) + \bar{H}_r(y, z) \\ &= \bar{a}_x (-\frac{1}{6\pi}) \cos 6z \cdot e^{j6\sqrt{3}y} \quad (\text{A/m}) \end{aligned}$$

P. 8-19 a) From Eqs. (8-80a) and (8-80b):

$$\bar{E}_i(x, z; t) = \bar{a}_y 2E_{i0} \sin(\beta_z z \cos \theta_i) \sin(\omega t - \beta_x x \sin \theta_i)$$

$$\begin{aligned} \bar{H}_i(x, z; t) = & \bar{a}_x \left(-2 \frac{E_{i0}}{\eta_1} \right) \cos \theta_i \cos(\beta_z z \cos \theta_i) \cos(\omega t - \beta_x x \sin \theta_i) \\ & + \bar{a}_z \left(2 \frac{E_{i0}}{\eta_1} \right) \sin \theta_i \sin(\beta_z z \cos \theta_i) \sin(\omega t - \beta_x x \sin \theta_i). \end{aligned}$$

$$b) \bar{P}_{av} = \frac{1}{2} \operatorname{Re} (\bar{E} \times \bar{H}^*) = \bar{a}_x \frac{2E_{i0}^2}{\eta_1} \sin \theta_i \sin^2(\beta_z z \cos \theta_i).$$

P. 8-20 a) From Eqs. (8-86a) and (8-86b):

$$\begin{aligned} \bar{E}_i(x, z; t) = & -2E_{i0} \left[\bar{a}_x \cos \theta_i \sin(\beta_z z \cos \theta_i) \cos(\omega t - \beta_x x \sin \theta_i) \right. \\ & \left. + \bar{a}_z \sin \theta_i \cos(\beta_z z \cos \theta_i) \sin(\omega t - \beta_x x \sin \theta_i) \right], \end{aligned}$$

$$\bar{H}_i(x, z; t) = +\bar{a}_y \frac{2E_{i0}}{\eta_1} \cos(\beta_z z \cos \theta_i) \sin(\omega t - \beta_x x \sin \theta_i).$$

$$b) \bar{P}_{av} = \frac{1}{2} \operatorname{Re} (\bar{E} \times \bar{H}^*) = \bar{a}_x \frac{2E_{i0}^2}{\eta_1} \sin \theta_i \cos^2(\beta_z z \cos \theta_i).$$

P. 8-21 $1 + \Gamma = \tau$, $|\Gamma| \leq 1$.

$$|\tau| = |\Gamma| \rightarrow \Gamma < 0 \rightarrow \eta_1 - \eta_2 = 2\eta_2.$$

$$\rightarrow \eta_1 = 3\eta_2 \rightarrow |\Gamma| = \frac{1}{2}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 3, \quad S_{dB} = 20 \log_{10} 3 = 9.54 \text{ (dB)}.$$

P. 8-22 a) In the lossy medium (medium 2):

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-\alpha_2 z} e^{-j\beta_2 z},$$

$$\text{where } \alpha_2 = \sqrt{\frac{\mu_2 \epsilon_2}{2}} \left[\sqrt{1 + \left(\frac{\sigma_2}{\omega \epsilon_2} \right)^2} - 1 \right]^{\frac{1}{2}}, \quad \beta_2 = \sqrt{\frac{\mu_2 \epsilon_2}{2}} \left[\sqrt{1 + \left(\frac{\sigma_2}{\omega \epsilon_2} \right)^2} + 1 \right]^{\frac{1}{2}}.$$

In air, $\beta_1 = 6 \text{ (rad/m)}$, $\omega = \beta_1 c = 1.8 \times 10^9 \text{ (rad/s)}$.

$$\tan \delta_c = \frac{\sigma_2}{\omega \epsilon_2} = 0.5 \rightarrow \alpha_2 = 2.30 \text{ (Np/m)}, \quad \beta_2 = 9.76 \text{ (rad)}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{120\pi}{\sqrt{\epsilon_2 (1 + \tan^2 \delta_c)}} = 225 e^{j13.3^\circ}$$

$$\therefore \bar{E}_t = \bar{a}_x E_{t0} e^{-2.30z} e^{-j9.76z}$$

$$\bar{H}_t = \bar{a}_z \times \frac{\bar{E}_t}{\eta_2} = \bar{a}_y \frac{E_{t0}}{225} e^{-j9.76z} e^{-2.30z} e^{j13.3^\circ}.$$

We also have $\bar{H}_i = \bar{a}_y \frac{10}{120\pi} e^{-j6z}$

Let $\bar{E}_r = \bar{a}_x E_{r0} e^{j6z} \longrightarrow \bar{H}_r = -\bar{a}_y \frac{E_{r0}}{120\pi} e^{j6z}$

Boundary conditions for \bar{E} and \bar{H} at $z=0$:

$$\begin{cases} 10 + E_{r0} = E_{t0} \\ 10 - E_{r0} = E_{t0} \sqrt{\epsilon_{r1}} (1 + \tan^2 \delta_c)^{1/4} e^{-j13.3^\circ} \end{cases}$$

$$E_{r0} = 2.77 e^{j157^\circ}, \quad E_{t0} = 7.53 e^{-j172^\circ}$$

$$\bar{E}_r(z,t) = \bar{a}_x 2.77 \cos(1.8 \times 10^9 t + 6z + 157^\circ) \quad (\text{V/m})$$

$$\bar{H}_r(z,t) = -\bar{a}_y 0.073 \cos(1.8 \times 10^9 t + 6z + 157^\circ) \quad (\text{A/m})$$

$$\bar{E}_t(z,t) = \bar{a}_x 7.53 e^{-2.30z} \cos(1.8 \times 10^9 t - 9.76z - 172^\circ) \quad (\text{V/m})$$

$$\bar{H}_t(z,t) = \bar{a}_y 0.033 e^{-2.30z} \cos(1.8 \times 10^9 t - 9.76z + 175^\circ) \quad (\text{A/m}).$$

$$b) (\bar{P}_{av})_1 = \bar{a}_z \left(\frac{10^2}{2 \times 120\pi} - \frac{2.77^2}{2 \times 120\pi} \right) = \bar{a}_z 0.122 \quad (\text{W/m}^2),$$

$$\begin{aligned} (\bar{P}_{av})_2 &= \bar{a}_z \frac{7.53^2}{2 \times 225} (\cos 13.3^\circ) e^{-4.61z} \\ &= \bar{a}_z 0.122 e^{-4.61z} \quad (\text{W/m}^2). \end{aligned}$$

P. 8-23 From Eqs. (8-107a), (8-107b), (8-108a), (8-108b), (8-109a) and (8-109b) we have

$$\bar{E}_1 = \bar{a}_x (E_{i0} e^{-j\beta_0 z} + E_{r0} e^{j\beta_0 z}), \quad \bar{H}_1 = \bar{a}_y \frac{1}{\eta_0} (E_{i0} e^{-j\beta_0 z} - E_{r0} e^{j\beta_0 z})$$

$$\bar{E}_2 = \bar{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}), \quad \bar{H}_2 = \bar{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

$$\bar{E}_t = \bar{a}_x E_{t0} e^{-j\beta_2 z}, \quad \bar{H}_t = \bar{a}_y \frac{1}{\eta_0} E_{t0} e^{-j\beta_2 z}.$$

Boundary conditions at $z=0$: $\bar{E}_1(0) = \bar{E}_2(0)$, $\bar{H}_1(0) = \bar{H}_2(0)$.

at $z=d$: $\bar{E}_2(d) = \bar{E}_t(d)$, $\bar{H}_2(d) = \bar{H}_t(d)$.

Four equations to solve four unknowns E_2^+ , E_2^- , E_{r0} , and E_{t0} in terms of E_{i0} :

$$a) E_{r0} = - \frac{j(\eta_0^2 - \eta_2^2) \tan \beta_2 d}{\eta_0 \eta_2 + j(\eta_0^2 + \eta_2^2) \tan \beta_2 d} E_{i0}$$

$$E_2^+ = \frac{\eta_2 (\eta_0 + \eta_2) e^{j\beta_2 d}}{\eta_0 \eta_2 \cos \beta_2 d + j(\eta_0^2 + \eta_2^2) \sin \beta_2 d} E_{i0}.$$

$$E_1^- = \frac{\eta_2 (\eta_0 - \eta_2) e^{-j\beta_2 d}}{\eta_0 \eta_2 \cos \beta_2 d + j(\eta_0^2 + \eta_2^2) \sin \beta_2 d} E_{i0}$$

$$E_{t0} = \frac{2\eta_0 \eta_2 e^{j\beta_2 d}}{\eta_0 \eta_2 \cos \beta_2 d + j(\eta_0^2 + \eta_2^2) \sin \beta_2 d} E_{i0}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi$, $\eta_2 = \sqrt{\mu_2/\epsilon_2}$.

$$\beta_0 = \omega/c, \quad \beta_2 = \omega\sqrt{\mu_2\epsilon_2}$$

b) If $d = \lambda_2/4$, $\beta_2 d = \pi/2$, $E_{r0} = -\frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2} E_{i0}$

$$\therefore \Gamma = -\frac{\eta_0^2 - \eta_2^2}{\eta_0^2 + \eta_2^2} \neq 0 \text{ unless } \eta_2 = \eta_0$$

$$\therefore \Gamma = 0 \text{ when } d = n\lambda_2/2, n = 1, 2, 3, \dots$$

P.8-24 a) From Example 8-10: $\eta_2 = \sqrt{\eta_1 \eta_3} \rightarrow \epsilon_{2r} = \sqrt{\epsilon_{r1} \epsilon_{r3}} = 2$.

Wavelength of red light in dielectric coating:

$$\lambda_2 = 0.75 \frac{u_{r1}}{c} = \frac{0.75}{\sqrt{\epsilon_{2r}}} = \frac{0.75}{\sqrt{2}} = 0.530 \text{ } (\mu\text{m})$$

$$d = \lambda_2/4 = 0.133 \text{ } (\mu\text{m}).$$

b) For violet light, $\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297 \text{ } (\mu\text{m})$.

$$\frac{d}{\lambda_2'} = 0.447 \rightarrow \beta_2 d = 0.894\pi.$$

From Eq. (8-116) and using impedances normalized with respect to $\eta_1 = \eta_0$:

$$\begin{aligned} Z_2(0) &= \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} = \frac{1}{\sqrt{2}} \frac{\frac{1}{2} + j\frac{1}{2} \tan \beta_2 d}{\frac{1}{2} + j\frac{1}{2} \tan \beta_2 d} \\ &= \frac{0.5 - j0.247}{1 - j0.247} \end{aligned}$$

$$\Gamma = \frac{Z_2(0) - 1}{Z_2(0) + 1} = 0.316 e^{j198^\circ}$$

$$\begin{aligned} \text{Percentage of incident reflected} &= |\Gamma|^2 \times 100\% \\ &= (0.316)^2 \times 100\% = 10\% \end{aligned}$$

P.8-25 $\Gamma_0 = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}$, $Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d}$

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \frac{\eta_1}{\eta_2} = \frac{1 - \Gamma_{12}}{1 + \Gamma_{12}}$$

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \longrightarrow \frac{\eta_2}{\eta_3} = \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}}$$

$$\begin{aligned} \Gamma_0 &= \frac{1 + j \frac{\eta_1}{\eta_2} \tan \beta_2 d - \frac{\eta_2}{\eta_1} \left(\frac{\eta_1}{\eta_2} + j \tan \beta_2 d \right)}{1 + j \frac{\eta_1}{\eta_3} \tan \beta_2 d + \frac{\eta_1}{\eta_2} \left(\frac{\eta_2}{\eta_3} + j \tan \beta_2 d \right)} \\ &= \frac{1 + j \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} \tan \beta_2 d - \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} \left(\frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} + j \tan \beta_2 d \right)}{1 + j \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} \tan \beta_2 d + \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} \left(\frac{1 - \Gamma_{23}}{1 + \Gamma_{23}} + j \tan \beta_2 d \right)} \\ &= \frac{(\Gamma_{12} + \Gamma_{23}) + j(\Gamma_{12} - \Gamma_{23}) \tan \beta_2 d}{(1 + \Gamma_{12} \Gamma_{23}) + j(1 - \Gamma_{12} \Gamma_{23}) \tan \beta_2 d} \end{aligned}$$

P. 8-26 $\bar{E}_1 = \bar{a}_x (E_{i0} e^{-j\beta_1 z} + E_{r0} e^{j\beta_1 z})$

$$\bar{H}_1 = \bar{a}_y \frac{1}{\eta_1} (E_{i0} e^{-j\beta_1 z} - E_{r0} e^{j\beta_1 z})$$

$$\bar{E}_2 = \bar{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z})$$

$$\bar{H}_2 = \bar{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

At $z = d$, $\bar{E}_2 = 0 \longrightarrow E_2^- = -E_2^+ e^{-j2\beta_2 d}$.

$$\bar{E}_2 = \bar{a}_x E_2^+ [e^{-j\beta_2 z} - e^{j\beta_2(z-2d)}]$$

$$\bar{H}_2 = \bar{a}_y \frac{E_2^+}{\eta_2} [e^{-j\beta_2 z} + e^{j\beta_2(z-2d)}]$$

Boundary conditions $E_1(0) = E_2(0) \longrightarrow E_{i0} + E_{r0} = E_2^+ (1 - e^{-j2\beta_2 d})$,
at $z = 0$: $H_1(0) = H_2(0) \longrightarrow E_{i0} - E_{r0} = E_2^+ \frac{\eta_2}{\eta_1} (1 + e^{-j2\beta_2 d})$.

$$E_2^+ = \frac{2\eta_2 E_{i0}}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2) e^{-j2\beta_2 d}}$$

$$E_{r0} = - \left(\frac{\eta_0 - j\eta_2 \tan \beta_2 d}{\eta_0 + j\eta_2 \tan \beta_2 d} \right) E_{i0}$$

a) $\bar{E}_r(x, t) = \bar{a}_x E_{i0} \cos \left[\omega \left(t - \frac{z}{u_p} \right) + \theta \right]$, $\theta = \pi - 2 \tan^{-1} \left(\frac{\eta_2 \tan \beta_2 d}{\eta_0} \right)$.

b) $\bar{E}_1(z, t) = \bar{a}_x E_{i0} \left\{ \cos \omega \left(t - \frac{z}{u_p} \right) + \cos \left[\omega \left(t - \frac{z}{u_p} \right) + \theta \right] \right\}$.

c) $\bar{E}_2(z, t) = \bar{a}_x \frac{2\eta_2 E_{i0}}{\sqrt{2[(\eta_0^2 + \eta_2^2) + (\eta_0^2 - \eta_2^2) \cos 2\beta_2 d]}} \left\{ \cos \left[\omega \left(t - \frac{z}{u_p} \right) + \psi \right] - \cos \left[\omega \left(t + \frac{z}{u_p} \right) - \frac{2\omega d}{u_p} + \psi \right] \right\}$,

$$\psi = \tan^{-1} \left[\frac{(\eta_0 - \eta_2) \sin 2\beta_2 d}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2) \cos 2\beta_2 d} \right]$$

$$d) (\bar{P}_{av})_1 = \frac{1}{2} \operatorname{Re} (\bar{E}_1 \times \bar{H}_1^*) = 0.$$

$$e) (\bar{P}_{av})_2 = 0.$$

$$f) \text{ Let } E_{r0} = -E_{i0} \longrightarrow \tan \beta_2 d = 0 \longrightarrow d = n\lambda_2/2, n=0,1,2,\dots$$

P. 8-27 $k_2 = \beta_2 - j\alpha_2 = (1-j)\frac{1}{\delta}, \quad \alpha_2 = \beta_2 = \frac{1}{\delta} = \sqrt{\pi f \mu_2 \epsilon_2}.$

$$\eta_2 = (1+j)\frac{\eta_0}{\sigma} \ll \eta_0 \quad \text{at } 10(\text{MHz}).$$

a) From Problem P. 8-23,

$$E_2^+ = \eta_2 H_2^+ \approx -j \left(\frac{\eta_0}{\eta_2} \right) \frac{e^{\alpha_2 d} e^{j\beta_2 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}$$

$$b) E_2^- = -\eta_2 H_2^- \approx -j \left(\frac{\eta_0}{\eta_2} \right) \frac{e^{-\alpha_2 d} e^{-j\beta_2 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}$$

$$c) E_{30} = E_{t0} = \eta_0 H_{30} \approx -j \left(\frac{\eta_0}{\eta_2} \right) \frac{2e^{j\beta_2 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}$$

$$d) E_{r0} \approx - \frac{E_{i0}}{1 - j \frac{\eta_0}{\eta_2} \cot(\beta_2 - j\alpha_2)d}$$

$$\approx - \left(1 + j \frac{\eta_0}{\eta_2} \frac{1 + j \tan \beta_2 d \tanh \alpha_2 d}{\tan \beta_2 d - j \tanh \alpha_2 d} \right) E_{i0}.$$

$$(\bar{P}_{av})_1 = \frac{1}{2} \operatorname{Re} [(\bar{E}_{i0} \times \bar{H}_{i0}^*) - (E_{r0} \times \bar{H}_{r0}^*)]$$

$$= \bar{a}_z \frac{\alpha}{\eta_0^2 \sigma} (A + B),$$

$$\text{where } \frac{1 + j \tan \beta_2 d \tanh \alpha_2 d}{\tan \beta_2 d - j \tanh \alpha_2 d} = A + jB.$$

$$(\bar{P}_{av})_1 = \frac{\alpha}{\eta_0^2 \sigma} \frac{\sin \beta_2 d \cos \beta_2 d + \sinh \alpha_2 d \cosh \alpha_2 d}{(\sin \beta_2 d \cosh \alpha_2 d)^2 + (\cos \beta_2 d \sinh \alpha_2 d)^2}$$

$$(\bar{P}_{av})_3 = \frac{1}{2\eta_0} |E_{30}|^2 = \frac{1}{\eta_0^2} \left(\frac{\alpha}{\sigma} \right)^2 \frac{4E_{i0}^2}{(\sin \beta_2 d \cosh \alpha_2 d)^2 + (\cos \beta_2 d \sinh \alpha_2 d)^2}$$

$$\therefore \frac{(\bar{P}_{av})_3}{(\bar{P}_{av})_1} = \frac{4}{\eta_0} \left(\frac{\alpha}{\sigma} \right) \frac{1}{\sin \beta_2 d \cos \beta_2 d + \sinh \alpha_2 d \cosh \alpha_2 d}$$

$$(\bar{P}_{av})_1 = \frac{1}{2\eta_0} E_{i0}^2.$$

$$\frac{(\bar{P}_{av})_3}{(\bar{P}_{av})_1} = \frac{8}{\eta_0^2} \left(\frac{\alpha}{\sigma} \right)^2 \frac{1}{(\sin \beta_2 d \cosh \alpha_2 d)^2 + (\cos \beta_2 d \sinh \alpha_2 d)^2}$$

$$\text{At } f = 10^7 (\text{Hz}), \quad \sigma = 5.90 \times 10^7 (\text{S/m}), \quad \alpha_2 = \beta_2 = 4.785 \times 10^4, \quad d = \delta = \frac{1}{\alpha_2}$$

$$\frac{(\bar{P}_{av})_3}{(\bar{P}_{av})_1} = 1.839 \times 10^{-11}.$$

P. 8-28 $k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2 \mu_0 \epsilon_2 - j\omega \mu_0 \sigma_2$. (1)

Continuity conditions at $z=0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega \sqrt{\mu_0 \epsilon_0} \sin \theta_i = \beta_x = 2.09 \times 10^4 \quad (2)$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z}. \quad (3)$$

Combining (1), (2) and (3), we can solve α_{2z} and β_{2z} in terms of ω , μ_0 , ϵ_2 , σ_2 , and β_x . But, since

$$\beta_x^2 \ll \omega^2 \mu_0 \epsilon_2,$$

we have $\alpha_{2z} - \alpha_{2z} \approx \beta_{2z} \approx \frac{1}{\delta} = \sqrt{\pi f \mu_0 \sigma_2} = 0.3974 \text{ (m}^{-1}\text{)}.$

a) $\theta_t = \tan^{-1} \frac{\beta_x}{\beta_{2z}} \approx \tan^{-1} \frac{2.09}{0.3974} \times 10^4 \approx 5.26 \times 10^{-4} \text{ (rad)}$
 $\approx 0.03^\circ.$

b) $\Gamma_1 = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_0 \cos \theta_i}$ $\eta_2 = \frac{\sigma_2}{\sigma_2} (1+j) = 0.0993(1+j).$
 $= \frac{2 \times 0.0993(1+j)}{0.0993(1+j) + 377 \cos 88^\circ}$ $\cos \theta_t = \cos 0.03^\circ \approx 1.$
 $\approx 0.0151(1+j) = 0.0214 e^{j\pi/4}$

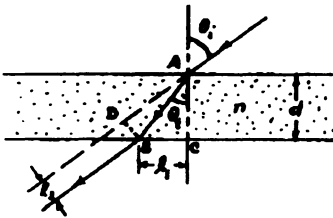
c) $(\rho_{av})_i = \frac{E_{i0}}{2\eta_0}$.

$$E_{t0} \approx 2E_{i0} \frac{\eta_2}{\eta_0}, \quad H_{t0} \approx \frac{2E_{i0}}{\eta_0} \rightarrow (\rho_{av})_t = 2 \frac{E_{i0}^2 \eta_2}{\eta_0^2 \sigma_2} e^{-2\alpha_{2z}}$$

$$\therefore \frac{(\rho_{av})_t}{(\rho_{av})_i} = \frac{4\eta_2}{\eta_0 \sigma_2} e^{-2\alpha_{2z}} = 1.054 \times 10^{-3} e^{-0.794z}$$

d) $20 \log_{10} e^{-\alpha_{2z}} = -30 \rightarrow z = \frac{1.5}{\alpha_{2z} \log_{10} e} = 8.69 \text{ (m)}.$

P. 8-29



a) Snell's law:

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{1}{n}$$

$$\theta_t = \sin^{-1} \left(\frac{1}{n} \sin \theta_i \right).$$

b) $\cos \theta_t = \sqrt{1 - \left(\frac{1}{n} \sin \theta_i \right)^2}.$

$$l_1 = \overline{BC} = \overline{AC} \tan \theta_t = d \frac{\sin \theta_t}{\cos \theta_t} = \frac{d \sin \theta_i}{\sqrt{n^2 - \sin^2 \theta_i}}$$

c) $l_2 = \overline{BD} = \overline{AC} \sin(\theta_i - \theta_t) = \frac{d}{\cos \theta_t} (\sin \theta_i \cos \theta_t - \cos \theta_i \sin \theta_t)$
 $= d \sin \theta_i \left[1 - \frac{\cos \theta_i}{\sqrt{n^2 - \sin^2 \theta_i}} \right].$

P. 8-30 a) $\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \rightarrow \sin \theta_i = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i > 1$ for $\theta_i > \theta_c$.
 $\cos \theta_t = -j \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right) \sin^2 \theta_i - 1}$

From Eqs. (8-135a) and (8-135b):

$$\vec{E}_t(x, z) = \bar{a}_y E_{t0} e^{-\alpha_2 z} e^{-j\beta_{2x} x},$$

$$\vec{H}_t(x, z) = \frac{E_{t0}}{\eta_2} (\bar{a}_x j \alpha_2 + \bar{a}_z \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i) e^{-\alpha_2 z} e^{-j\beta_{2x} x},$$

where $\beta_{2x} = \beta_2 \sin \theta_t = \beta_2 \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$,

$$\alpha_2 = \beta_2 \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right) \sin^2 \theta_i - 1},$$

$$E_{t0} = \frac{2\eta_2 \cos \theta_i E_{i0}}{\eta_2 \cos \theta_i - j\eta_1 \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right) \sin^2 \theta_i - 1}} \quad \text{from Eq. (8-139)}$$

b) $(\mathcal{P}_{av})_{zx} = \frac{1}{2} \operatorname{Re} (E_{ty} H_{tx}^*) = 0.$

P. 8-31 a) $\theta_c = \sin^{-1} \sqrt{\epsilon_2 / \epsilon_1} = \sin^{-1} \sqrt{1/81} = 6.38^\circ.$

b) $\theta_i = 20^\circ > \theta_c$. $\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i = 3.08$
 $\cos \theta_t = -j \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right) \sin^2 \theta_i - 1} = -j 2.91$

$$\Gamma_{\perp} = \frac{\sqrt{\epsilon_2} \cos \theta_i - \cos \theta_t}{\sqrt{\epsilon_2} \cos \theta_i + \cos \theta_t} = e^{j38^\circ} = e^{j0.66}$$

c) $\tau_{\perp} = \frac{2\sqrt{\epsilon_2} \cos \theta_i}{\sqrt{\epsilon_2} \cos \theta_i + \cos \theta_t} = 1.89 e^{j19^\circ} = 1.89 e^{j0.33}$

d) The transmitted wave in air varies as

$$e^{-\alpha_2 z} e^{-j\beta_{2x} x},$$

where $\alpha_2 = \beta_2 \sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right) \sin^2 \theta_i - 1} = \frac{2\pi}{\lambda_0} (2.91).$

Attenuation in air for each wavelength

$$= 20 \log_{10} e^{-\alpha_2 \lambda_0} = 159 \text{ (dB)}.$$

P. 8-32 When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0$, $\tau_{\perp} = \frac{2\eta_2}{\eta_1 + \eta_0}$.

$$\frac{(\mathcal{P}_{av})_{t1}}{(\mathcal{P}_{av})_i} = \frac{\eta_0}{\eta_2} \tau_{\perp}^2 = \frac{4\eta_0 \eta_2}{(\eta_1 + \eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45^\circ > \theta_c = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ.$$

On exit from the prism, $\tau_2 = \frac{2\eta_0}{\eta_2 + \eta_0}$.

$$\frac{(\rho_{av})_o}{(\rho_{av})_i} = \frac{\eta_0}{\eta_0} \tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2 + \eta_0)^2}.$$

$$\therefore \frac{(\rho_{av})_o}{(\rho_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2 + \eta_0)^2} \right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{(1 + \sqrt{\epsilon_r})^2} \right]^2 = 0.79.$$

P. 8-33 a) For perpendicular polarization and $\mu_1 \neq \mu_2$:

$$\sin \theta_{BL} = \frac{1}{\sqrt{1 + \left(\frac{\mu_1}{\mu_2}\right)^2}}$$

Under condition of no reflection:

$$\begin{aligned} \cos \theta_t &= \sqrt{1 - \frac{\eta_1^2}{\eta_2^2} \sin^2 \theta_{BL}} = \frac{1}{\sqrt{1 + \left(\frac{\mu_1}{\mu_2}\right)^2}} \\ &= \sin \theta_{BL} \longrightarrow \theta_t + \theta_{BL} = \pi/2. \end{aligned}$$

b) For parallel polarization and $\epsilon_1 \neq \epsilon_2$:

$$\begin{aligned} \sin \theta_{BL} &= \frac{1}{\sqrt{1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}} \\ \cos \theta_t &= \sqrt{1 - \frac{\eta_1^2}{\eta_2^2} \sin^2 \theta_{BL}} = \frac{1}{\sqrt{1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}} \\ &= \sin \theta_{BL} \longrightarrow \theta_t + \theta_{BL} = \pi/2. \end{aligned}$$

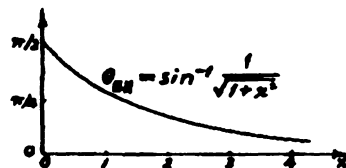
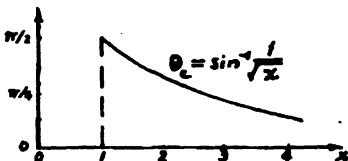
P. 8-34 a) $\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$; $\sin \theta_{BL} = \frac{1}{\sqrt{1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}$



$$\longrightarrow \tan \theta_{BL} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

$$\therefore \sin \theta_c = \tan \theta_{BL} \quad (\theta_c > \theta_{BL})$$

b) Let $\epsilon_1/\epsilon_2 = x$.



P. 8-35 a) For perpendicular polarization:

$$\Gamma_{\perp} = \frac{\sqrt{\epsilon_{r2}} \cos \theta_i - \sqrt{\epsilon_{r1}} \cos \theta_t}{\sqrt{\epsilon_{r2}} \cos \theta_i + \sqrt{\epsilon_{r1}} \cos \theta_t}$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} \sin \theta_i, \quad \cos \theta_t = \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right) \sin^2 \theta_i}$$

$$\Gamma_{\perp} = \frac{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cos \theta_i - \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2 \theta_i}}{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cos \theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2 \theta_i}}$$

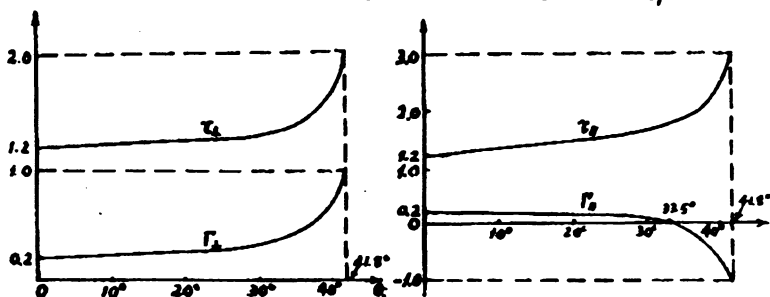
$$\tau_{\perp} = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} = \frac{2 \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cos \theta_i}{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cos \theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2 \theta_i}}$$

For parallel polarization:

$$\Gamma_{\parallel} = \frac{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right) \sin^2 \theta_i} - \cos \theta_t}{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right) \sin^2 \theta_i} + \cos \theta_t}$$

$$\tau_{\parallel} = \frac{2 \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cos \theta_i}{\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right) \sin^2 \theta_i} + \cos \theta_t}$$

b) $\epsilon_{r1}/\epsilon_{r2} = 2.25$, $\sqrt{\epsilon_{r1}/\epsilon_{r2}} = 1.5 \rightarrow \theta_c = \sin^{-1} \sqrt{\frac{\epsilon_1}{\epsilon_2}} = 41.8^\circ$



P. 8-36 a) $\Gamma'_{\parallel} = \frac{(E_r)_{\tan}}{(E_i)_{\tan}} \Big|_{z=0} = \frac{E_{r2} \cos \theta_t}{E_{i0} \cos \theta_i} = \frac{E_{r2}}{E_{i0}} = \Gamma'_{\parallel} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$

$$\tau'_{\parallel} = \frac{(E_t)_{\tan}}{(E_i)_{\tan}} \Big|_{z=0} = \frac{E_{t2} \cos \theta_t}{E_{i0} \cos \theta_i} = \tau_{\parallel} \left(\frac{\cos \theta_t}{\cos \theta_i} \right) = \frac{2 \eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

We have $1 + \Gamma'_{\parallel} = \tau'_{\parallel}$.

which compares with Eq. (8-151):

$$1 + \Gamma_{\parallel} = \tau_{\parallel}$$

Chapter 9

P. 9-1

$$\bar{\nabla} \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ H_x & 0 & 0 \end{vmatrix} = \bar{a}_y j\omega\epsilon E_y \longrightarrow \frac{\partial H_x}{\partial y} = 0.$$

$$\bar{\nabla} \times \bar{E} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta \\ 0 & E_y & 0 \end{vmatrix} = -\bar{a}_x j\omega\mu H_x \longrightarrow \frac{\partial E_x}{\partial x} = 0.$$

P. 9-2 a) $\bar{\nabla} \times (\bar{a}_x E_x + \bar{a}_y E_y) = -j\omega\mu(\bar{a}_x H_x + \bar{a}_y H_y)$

$$\longrightarrow \begin{cases} \beta E_y = -\omega\mu H_x & \textcircled{1} \\ \beta E_x = \omega\mu H_y & \textcircled{2} \\ \frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial y} & \textcircled{3} \end{cases}$$

$$\nabla \times (a_x H_x + a_y H_y) = j\omega\epsilon (a_x E_x + a_y E_y)$$

$$\longrightarrow \begin{cases} \beta H_y = \omega\epsilon E_x & \textcircled{4} \\ \beta H_x = -\omega\epsilon E_y & \textcircled{5} \\ \frac{\partial H_x}{\partial x} = \frac{\partial H_x}{\partial y} & \textcircled{6} \end{cases}$$

From $\textcircled{1}$ and $\textcircled{4}$: $\beta = \omega\sqrt{\mu\epsilon}$ $\textcircled{7}$

From $\textcircled{2}$ or $\textcircled{5}$: $\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}} = \eta$ $\textcircled{8}$

From $\textcircled{1}$ or $\textcircled{4}$: $\frac{E_y}{H_x} = -\sqrt{\frac{\mu}{\epsilon}} = -\eta$ $\textcircled{9}$

b) From $\textcircled{1}$: $\frac{\partial^2 E_y}{\partial y \partial x} = \frac{\partial^2 E_x}{\partial y^2}$ $\textcircled{10}$

From $\textcircled{2}$, $\textcircled{3}$, and $\textcircled{10}$: $\frac{\partial E_x}{\partial x} = -\frac{\partial E_x}{\partial y} \longrightarrow \frac{\partial^2 E_x}{\partial x^2} = -\frac{\partial^2 E_x}{\partial x \partial y}$ $\textcircled{11}$

Combining $\textcircled{10}$ and $\textcircled{11}$, we have $\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} = 0$.

Similarly, $\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} = 0$.

P. 9-3 Eq. (9-20): $Z_0 = \frac{d}{w} \sqrt{\frac{\mu}{\epsilon}}$.

a) $Z_0 = \frac{d'}{w} \sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w} \sqrt{\frac{\mu}{\epsilon}} \longrightarrow d' = \sqrt{2} d$.

b) $Z_0 = \frac{d}{w'} \sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w} \sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = \frac{1}{\sqrt{2}} w$.

c) $Z_0 = \frac{2d}{w'} \sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w} \sqrt{\frac{\mu}{\epsilon}} \longrightarrow w' = 2w$.

d) $u_p = \frac{1}{\sqrt{\mu\epsilon}} \longrightarrow \begin{cases} u_{pa} = u_p / \sqrt{2} \text{ for case a.} \\ u_{pb} = u_p / \sqrt{2} \text{ for case b.} \\ u_{pc} = u_p \text{ for case c.} \end{cases}$

$$\begin{aligned}
 \underline{P.9-7} \quad \gamma &= j\omega\sqrt{LC} \left(1 - j\frac{R}{\omega L}\right)^{1/2} \left(1 - j\frac{G}{\omega C}\right)^{1/2} \\
 &\cong j\omega\sqrt{LC} \left[1 - j\frac{R}{2\omega L} + \frac{1}{8}\left(\frac{R}{\omega L}\right)^2 + j\frac{R^3}{16\omega^3 L^3}\right] \\
 &\quad \times \left[1 - j\frac{G}{2\omega C} + \frac{1}{8}\left(\frac{G}{\omega C}\right)^2 + j\frac{G^3}{16\omega^3 C^3}\right] = \alpha + j\beta
 \end{aligned}$$

$$\begin{aligned}
 \longrightarrow \alpha &= \frac{\sqrt{LC}}{2} \left(\frac{R}{L} + \frac{G}{C}\right) \left[1 - \frac{1}{8\omega^2} \left(\frac{R}{L} - \frac{G}{C}\right)^2\right] \\
 \beta &= \omega\sqrt{LC} \left[1 + \frac{1}{8\omega^2} \left(\frac{R}{L} - \frac{G}{C}\right)^2\right].
 \end{aligned}$$

$$\begin{aligned}
 Z_0 &= \sqrt{\frac{L}{C}} \left(1 - j\frac{R}{\omega L}\right)^{1/2} \left(1 - j\frac{G}{\omega C}\right)^{-1/2} \\
 &\cong \sqrt{\frac{L}{C}} \left[1 - j\frac{R}{2\omega L} + \frac{1}{8}\left(\frac{R}{\omega L}\right)^2\right] \left[1 - j\frac{G}{2\omega C} + \frac{1}{8}\left(\frac{G}{\omega C}\right)^2\right]^{-1/2} = R_0 + jX_0
 \end{aligned}$$

$$\longrightarrow R_0 = \sqrt{\frac{L}{C}} \left[1 + \frac{1}{8\omega^2} \left(\frac{R}{L} - \frac{G}{C}\right) \left(\frac{R}{L} + \frac{2G}{C}\right)\right]$$

$$X_0 = -\frac{1}{2\omega} \sqrt{\frac{L}{C}} \left(\frac{R}{L} - \frac{G}{C}\right).$$

$$u_p = \frac{\omega}{\beta} \cong \frac{1}{\sqrt{LC}} \left[1 - \frac{1}{8\omega^2} \left(\frac{R}{L} - \frac{G}{C}\right)^2\right].$$

$$\underline{P.9-8} \quad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{RG} \left(1 + \frac{j\omega L}{R}\right)^{1/2} \left(1 - \frac{j\omega C}{G}\right)^{1/2} = \alpha + j\beta$$

$$\longrightarrow \alpha \cong \sqrt{RG} \left[1 + \frac{1}{8} \left(\frac{\omega L}{R} - \frac{\omega C}{G}\right)^2\right], \quad \beta \cong \frac{\omega}{2} \sqrt{RG} \left(\frac{L}{R} + \frac{C}{G}\right).$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R}{G}} \left(1 + j\frac{\omega L}{R}\right)^{1/2} \left(1 + j\frac{\omega C}{G}\right)^{-1/2} = R_0 + jX_0$$

$$\longrightarrow R_0 \cong \sqrt{\frac{R}{G}} \left\{1 + \frac{\omega^2}{8} \left[\left(\frac{L}{R}\right)^2 + \frac{2LC}{RG} - 3\left(\frac{C}{G}\right)^2\right]\right\}$$

$$X_0 = \frac{\omega}{2} \sqrt{\frac{R}{G}} \left(\frac{L}{R} - \frac{C}{G}\right).$$

$$\underline{P.9-9} \quad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{Im}(Z_0) = 0 \longrightarrow \frac{R}{L} = \frac{G}{C} = 2 \quad (1)$$

$$\text{From Eqs. (9-49a), (9-49b), and (9-51a):} \quad \alpha = R\sqrt{\frac{C}{L}} \quad (2)$$

$$\text{Given: } Z_0 = 50 + j0 \text{ } (\Omega) \quad \beta = \omega\sqrt{LC} \quad (3)$$

$$\alpha = 0.01 \text{ (dB/m)} = 0.00115 \text{ (Np/m)} \quad Z_0 = \sqrt{\frac{L}{C}} \quad (4)$$

$$\beta = 0.8\pi \text{ (rad/m)}$$

$$f = 10^8 \text{ (Hz)}$$

$$R = \alpha Z_0 = 0.0576 \text{ } (\Omega/\text{m}), \quad L = \frac{\beta Z_0}{2\pi f} = 0.20 \text{ } (\mu\text{H}/\text{m})$$

$$G = \frac{\alpha^2}{R} = 23. \text{ } (\mu\text{S}/\text{m}), \quad C = \frac{\beta}{2\pi f} = 80 \text{ } (\text{pF}/\text{m}).$$

P.9-10 a) For two-wire transmission line:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \cosh^{-1} \left(\frac{D}{2a}\right) = \frac{120}{\pi} \ln \left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1} \right] = 330.$$

$$\frac{D}{2a} = 21.27 \longrightarrow D = 25.5 \times 10^{-3} \text{ (m).}$$

b) For coaxial transmission line:

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) = \frac{40}{\sqrt{\epsilon}} \ln\left(\frac{b}{a}\right) = 75$$

$$\frac{b}{a} = 6.52 \longrightarrow b = 3.91 \times 10^{-3} \text{ (m).}$$

$$\text{P. 9-11 } (P_{av})_L = (P_{av})_i = \frac{1}{2} \operatorname{Re} [V_i I_i^*]$$

$$V_i = \frac{Z_i}{Z_g + Z_i} V_g$$

$$= \frac{|V_g|^2 R_i}{(R_g + R_i)^2 + (X_g + X_i)^2}$$

$$I_i = \frac{V_g}{Z_g + Z_i}$$

$$\text{To maximize } (P_{av})_L, \text{ set } \left. \begin{array}{l} \frac{\partial (P_{av})_L}{\partial R_i} = 0 \\ \text{and } \frac{\partial (P_{av})_L}{\partial X_i} = 0 \end{array} \right\} \begin{array}{l} R_i = R_g, X_i = -X_g \\ \text{or } Z_i = Z_g^* \end{array}$$

$$\text{Max. } (P_{av})_L = \frac{|V_g|^2}{4R_g} = (P_{av})_{x_g}$$

\longrightarrow Max. power-transfer efficiency = 50%.

$$\text{P. 9-12 } V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\text{At } z=0: V(0) = V_i = V_0^+ + V_0^-, \quad I(0) = I_i = I_0^+ + I_0^- = \frac{1}{Z_0} (V_0^+ - V_0^-)$$

$$\longrightarrow V_0^+ = \frac{1}{2} (V_i + I_i Z_0), \quad V_0^- = \frac{1}{2} (V_i - I_i Z_0).$$

$$\text{a) } V(z) = \frac{1}{2} (V_i + I_i Z_0) e^{-\gamma z} + \frac{1}{2} (V_i - I_i Z_0) e^{\gamma z}$$

$$I(z) = \frac{1}{2Z_0} (V_i + I_i Z_0) e^{-\gamma z} - \frac{1}{2Z_0} (V_i - I_i Z_0) e^{\gamma z}.$$

$$\text{b) } V(z) = V_i \cosh \gamma z - I_i Z_0 \sinh \gamma z$$

$$I(z) = I_i \cosh \gamma z - \frac{V_i}{Z_0} \sinh \gamma z.$$

$$\text{P. 9-13 a) } -\frac{dV}{dz} = RI, \quad -\frac{dI}{dz} = GV$$

$$\left\{ \begin{array}{l} \frac{d^2 V}{dz^2} = -RGV \\ \frac{d^2 I}{dz^2} = -RGI \end{array} \right.$$

$$\text{b) } V(z) = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}$$

$$I(z) = I_0^+ e^{-\alpha z} + I_0^- e^{\alpha z}, \quad \alpha = \sqrt{RG}$$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = R_0 = \sqrt{\frac{R}{G}}$$

We have $V(z) = \frac{1}{2}(V_i + I_i R_0) e^{-\alpha z} + \frac{1}{2}(V_i - I_i R_0) e^{+\alpha z}$

$$I(z) = \frac{1}{2} \left(\frac{V_i}{R_0} + I_i \right) e^{-\alpha z} - \frac{1}{2} \left(\frac{V_i}{R_0} - I_i \right) e^{+\alpha z}$$

where $V_i = \frac{R_i}{R_0 + R_i} V_0$ and $I_i = \frac{V_0}{R_0 + R_i}$.

c) For an infinite line, $R_i = R_0$:

$$V(z) = \frac{R_0}{R_0 + R_0} V_0 e^{-\alpha z}, \quad I(z) = \frac{V_0}{R_0 + R_0} e^{-\alpha z}$$

d) For a finite line of length l terminated in R_L :

$$R_i = R_0 \frac{R_L + R_0 \tanh \alpha l}{R_0 + R_L \tanh \alpha l}$$

P.9-14 Distortionless line: $R_0 = \sqrt{\frac{L}{C}} = 50 (\Omega)$, $R = 0.5 (\Omega/m)$

$$\tan \left(\frac{\theta_L}{\omega C} \right) = \tan \left(\frac{G}{\omega C} \right) = 0.0018$$

$$\rightarrow \frac{G}{\omega C} = 1.79999 \times 10^{-3} \text{ (many digits necessary to obtain accurate answer for phase shift in part b)}$$

$$\frac{G}{C} = 8000\pi (1.79999 \times 10^{-3}) = 45.24 = \frac{R}{L}$$

$$L = \frac{R}{G/C} = 0.1105 \text{ (H/m)}, \quad C = \frac{L}{R_0^2} = 4.421 \text{ (\mu F/m)}$$

$$\alpha = \frac{R}{R_0} = 0.01 \text{ (Np/m)}, \quad \beta = \omega \sqrt{LC} = 5.555643 \text{ (rad/m)}$$

a) $V(z) = \frac{V_0 R_0}{R_0 + Z_0} e^{-\alpha z} e^{-j\beta z} = \frac{50}{9 + j3} e^{-0.01z} e^{-j\beta z}$

$$I(z) = V(z)/50$$

$$\therefore V(z, t) = 5.27 e^{-0.01z} \sin(8000\pi t - 5.555643z - 0.1024\pi) \quad (V)$$

$$I(z, t) = 0.105 e^{-0.01z} \sin(8000\pi t - 5.555643z - 0.1024\pi) \quad (A)$$

b) At $z = 5 \times 10^4$ (m), we obtain $V(5 \times 10^4, t)$ and $I(5 \times 10^4, t)$.

c) $(P_{av})_L = \frac{1}{2} R_0 [V_i I_i^*] \rightarrow$ Very very small.

{ Note: The given line length 50 (km) in the problem is a misprint. It should have been 50 (m), which would make the numbers more meaningful. }

P.9-15 a) From Eq. (9-97): $Z_{i1} = Z_0 \tanh \gamma l \approx \gamma l$.

From Eqs. (9-37) and (9-41): $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\therefore Z_{i1} = (R+j\omega L)l.$$

b) From Eq. (9-96): $Z_{i0} = Z_0 \coth \gamma l \approx \frac{Z_0}{\gamma l} = \frac{G-j\omega C}{[G^2+(\omega C)^2]}$

P.9-16 $\beta l = \frac{2\pi f}{c} l = \frac{8\pi}{3} = 480^\circ$.

$$\tan \beta l = \tan 480^\circ = -1.732.$$

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 50 \frac{(40+j30) + j50(-1.732)}{50 + j(40+j30)(-1.732)}$$

$$= 26.3 - j9.87 \text{ } (\Omega).$$

P.9-17 Given: $Z_{i0} = Z_0 \coth \gamma l = 250 \angle -50^\circ \text{ } (\Omega)$

$$Z_{i1} = Z_0 \tan \gamma l = 360 \angle 20^\circ \text{ } (\Omega).$$

a) $Z_0 = \sqrt{Z_{i0} Z_{i1}} = 300 \angle -15^\circ = 289.8 - j77.6 \text{ } (\Omega).$

$$\tanh \gamma l = \sqrt{\frac{Z_{i1}}{Z_{i0}}} = 1.2 \angle 35^\circ = 0.983 + j0.688 = \tanh(\alpha l + j\beta l)$$

$$l = 4 \text{ (m)} \longrightarrow \alpha = 0.139 \text{ (Np/m)}$$

$$\beta = 0.235 + \frac{n\pi}{4} \text{ (rad/m).}$$

($n=0$ in order to ensure $+R, L, G \& C$)

b) $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}, \quad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\longrightarrow R+j\omega L = \gamma Z_0; \quad G+j\omega C = \frac{\gamma}{Z_0}.$$

$$\omega = \beta c = 0.235 \times 3 \times 10^8 = 0.705 \times 10^8 \text{ (rad/m).}$$

We obtain $R = 58.6 \text{ } (\Omega), \quad L = 0.812 \text{ } (\mu\text{H/m})$

$$G = 0.246 \text{ (mS/m), } C = 12.4 \text{ (pF/m).}$$

P.9-18 a) For a lossless quarter-wave line section:

$$Z_i = \frac{R_0^2}{Z_L} = \frac{R_0^2}{R_L + jX_L} = \frac{R_0^2 R_L}{R_L^2 + X_L^2} - j \frac{R_0^2 X_L}{R_L^2 + X_L^2} = R'_i + jX'_i$$

$$\longrightarrow R'_i = \frac{R_0^2 R_L}{R_L^2 + X_L^2} \quad (1); \quad X'_i = - \frac{R_0^2 X_L}{R_L^2 + X_L^2} \quad (2)$$

(Resistance R'_i and capacitive reactance X'_i in series.)

Input impedance Z_i can also be expressed in terms of a resistance R_i and a capacitive reactance X_i in parallel:

$$Z_i = \frac{jX_i R_i}{R_i + jX_i} = \frac{R_i X_i^2}{R_i^2 + X_i^2} + j \frac{R_i^2 X_i}{R_i^2 + X_i^2} = R_i' + jX_i' \quad (3)$$

Combining Eqs. (1), (2), and (3), we find

$$R_i = \frac{R_o^2}{R_L} \quad \text{and} \quad X_i = -\frac{R_o^2}{X_L},$$

both of which are reminiscent of Eq. (9-94).

b) From Eq. (9-80a), $V(z') = I_L (Z_L \cos \beta z' + R_o \sin \beta z')$.

At the input, $z' = \lambda/4$, $\beta z' = \pi/2$, we have

$$V_i = V(\lambda/4) = I_L R_o.$$

At the load, $z' = 0$, $\beta z' = 0$, and $V_L = V(0) = I_L Z_L$.

$$\frac{|V_i|}{|V_L|} = \frac{R_o}{|Z_L|} = \frac{R_o}{\sqrt{R_L^2 + X_L^2}}$$

P 9-19 a) $|r| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_o} - 1\right|}{\left|\frac{Z_L}{Z_o} + 1\right|} = \frac{\sqrt{(r_L-1)^2 + x_L^2}}{\sqrt{(r_L+1)^2 + x_L^2}}$

where $r_L = R_L/Z_o$ and $x_L = X_L/Z_o$.

$$\longrightarrow x_L = \pm \left[\frac{\left(\frac{S-1}{S+1}\right)^2 (r_L+1)^2 - (r_L-1)^2}{1 - \left(\frac{S-1}{S+1}\right)^2} \right]^{1/2}$$

When $S=3$, $x_L = \pm \sqrt{(10r_L - 3r_L^2 - 3)/3}$.

b) $S=3$ and $r_L = 150/75 = 2 \longrightarrow x_L = \pm \sqrt{5/3}$.

$$X_L = x_L Z_o = \pm 96.8 \Omega.$$

c) From Eq. (9-114): $r_L + jx_L = \frac{r_m + jt}{1 + jr_m t}$, where $r_m = \frac{R_o}{Z_o}$ and $t = \tan \beta l_m$.

$$\longrightarrow r_m = \frac{(1+r_L^2+x_L^2) \pm \sqrt{(1+r_L^2+x_L^2)^2 - 4r_L^2}}{2r_L}$$

$$= 3 \text{ or } \frac{1}{3}, \text{ for } r_L=2 \text{ and } x_L^2 = 5/3.$$

Also, $x_L = \frac{(1-r_m^2)t}{1+r_m^2 t^2} \longrightarrow t = \frac{1}{2x_L r_m} \left[(1-r_m^2) \pm \sqrt{(1-r_m^2)^2 - 4x_L^2 r_m^2} \right]$

For $r = \frac{1}{3}$, $t = \begin{cases} 3\sqrt{3}/5 \longrightarrow l_m = 0.1865\lambda \\ \text{or } \sqrt{15} \longrightarrow l_m = 0.2098\lambda \end{cases}$

Use $l_m = 0.2098\lambda$ to obtain V_{min} nearest to the load at -0.2098λ .

P. 9-20 a) $|\Gamma|^2 = \left| \frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L} \right|^2 = \frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$

$$\frac{\partial |\Gamma|^2}{\partial Z_0} = 0 \longrightarrow Z_0 = \sqrt{R_L^2 + X_L^2}$$

If $Z_L = 40 + j30 (\Omega)$, $Z_0 = 50 (\Omega)$.

b) $\text{Min. } |\Gamma| = \sqrt{\frac{Z_0 - R_L}{Z_0 + R_L}} = \sqrt{\frac{50 - 40}{50 + 40}} = \frac{1}{3}$

$$\text{Min. } S = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$$

c) From Eq. (9-114): $r_m + jx_m = \frac{r_m + jx_m}{1 + j\beta r_m t} = 0.8 + j0.6$

$$\longrightarrow t = \frac{1}{2x_m r_m} \left[(1 - r_m^2) \pm \sqrt{(1 - r_m^2)^2 - 4x_m^2 r_m^2} \right] \quad \left(\begin{array}{l} \text{See problem} \\ \text{p. 9-19.} \end{array} \right)$$

At voltage minimum, $r_m = \frac{1}{S} = \frac{1}{2}$

$$t = 1 \quad (\text{Use negative sign.})$$

$$\tan \beta \ell_m = \tan(2\pi \ell_m / \lambda) = 1 \longrightarrow \ell_m = \frac{\lambda}{8}$$

\therefore Voltage minimum nearest to the load is $(\frac{\lambda}{2} - \frac{\lambda}{8})$
or $3\lambda/8$ from the load.

P. 9-21 a) From Eqs. (9-100a) and (9-101):

$$V(z') = \frac{Z_0}{2} (Z_L + Z_0) e^{-\gamma z'} [1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}]$$

where

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\phi}, \quad \phi = \theta_r - 2\beta z'$$

$$\text{Max. } |V(z')| = \left| \frac{Z_0}{2} (Z_L + Z_0) e^{-\alpha z'} \right| [1 + |\Gamma| e^{-2\alpha z'}] \quad \text{for } \phi = 0$$

$$\text{min. } |V(z')| = \left| \frac{Z_0}{2} (Z_L + Z_0) e^{-\alpha z'} \right| [1 - |\Gamma| e^{-2\alpha z'}] \quad \text{for } \phi = \pi$$

$$S(z') = \frac{\text{Max. } |V(z')|}{\text{min. } |V(z')|} = \frac{1 + |\Gamma| e^{-2\alpha z'}}{1 - |\Gamma| e^{-2\alpha z'}} \quad \left(\begin{array}{l} \text{There is a slight} \\ \text{ambiguity in } z' \text{ here,} \\ \text{which is insignificant} \\ \text{when } \alpha \text{ is small.} \end{array} \right)$$

b) From Eq. (9-132): $Z_i(z') = \frac{1 + |\Gamma| e^{-2\alpha z'} e^{j\phi}}{1 - |\Gamma| e^{-2\alpha z'} e^{j\phi}} Z_0$

At a voltage max., $\phi = 0$, $Z_i(z') = S(z') Z_0$.

c) At a voltage min., $\phi = \pi$, $Z_i(z') = \frac{Z_0}{S(z')}$

9-22 $Z_i = R'_o \frac{Z_L + jR'_o t}{R'_o + jZ_L t} \longrightarrow Z_L = R'_o \frac{Z_i - jR'_o t}{R'_o - jZ_i t}$

With $Z_i = 50 (\Omega)$ and $Z_L = 40 + j10 (\Omega)$, we have

$$40 + j10 = R'_o \frac{50 - jR'_o t}{R'_o - j50t} \longrightarrow \begin{cases} 40R'_o + 500t = 50R'_o \\ 10R'_o - 2000t = -R'^2_o t \end{cases}$$

$\therefore R'_o = 38.73 (\Omega)$.

$t = \tan \beta l' = 0.7746 \longrightarrow l' = 0.1049 \lambda$.

9-23 a) $|\Gamma| = \frac{5-1}{5+1} = \frac{2-1}{2+1} = \frac{1}{3}$.

From Eqs. (9-100a) and (9-101):

$$V(z') = \frac{I_0}{2} (Z_L + Z_o) e^{j\beta z'} [1 + |\Gamma| e^{j\phi}];$$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| e^{j\phi}, \quad \phi = \theta_r - 2\beta z'.$$

$V(z')$ is a minimum when $\phi = \pm \pi \longrightarrow \theta_r = 2 \left(\frac{2\pi}{\lambda} \right) = 0.3\lambda - \pi$

$$\therefore \Gamma = \frac{1}{3} e^{j0.2\pi} = 0.2\pi.$$

b) $Z_L = Z_o \left(\frac{1+\Gamma}{1-\Gamma} \right) = 466 + j206 (\Omega)$.

c) Terminating resistance $R_m = \frac{R_o}{S} = \frac{300}{2} = 150 (\Omega)$,

$$l_m = \frac{\lambda}{2} - z'_m = (0.5 - 0.3)\lambda = 0.2\lambda.$$

9-24 Eq. (9-114): $R_i + jX_i = R_o \frac{R_m + jR_o \tan \beta l_m}{R_o + jR_m \tan \beta l_m}$

Let $r_i = \frac{R_i}{R_o}$, $x_i = \frac{X_i}{R_o}$, $r_m = \frac{R_m}{R_o}$, and $t = \tan \beta l_m$.

$$r_i + jx_i = \frac{r_m + jt}{1 + jr_m t} \longrightarrow \begin{cases} r_m (1 + x_i t) = r_i \\ t(1 - r_m r_i) = x_i \end{cases}$$

We have

$$r_m = \frac{1}{2r_i} \left[(1 + r_i^2 + x_i^2) \pm \sqrt{(1 + r_i^2 + x_i^2)^2 - 4r_i^2} \right].$$

$$t = \frac{1}{2x_i} \left\{ -[1 - (r_i^2 + x_i^2)] \pm \sqrt{[1 - (r_i^2 + x_i^2)]^2 + 4x_i^2} \right\}.$$

$$l_m = \frac{\lambda}{2\pi} \tan^{-1} t.$$

P.9-25 $Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$

$$\Gamma = |\Gamma| e^{j\theta} \quad |\Gamma| = \frac{S-1}{S+1} \quad \theta_r = \frac{4\pi z'_n}{\lambda} \pm \pi$$

$$\begin{aligned} \therefore Z_L &= Z_0 \frac{(S+1) - (S-1) e^{j(4\pi z'_n/\lambda)}}{(S+1) + (S-1) e^{j(4\pi z'_n/\lambda)}} \\ &= Z_0 \frac{(S+1) e^{-j(2\pi z'_n/\lambda)} - (S-1) e^{j(2\pi z'_n/\lambda)}}{(S+1) e^{-j(2\pi z'_n/\lambda)} + (S-1) e^{j(2\pi z'_n/\lambda)}} \\ &= Z_0 \frac{1-jS \tan(2\pi z'_n/\lambda)}{S-j \tan(2\pi z'_n/\lambda)} \end{aligned}$$

P.9-26 a) Given: $V_g = 0.1 \angle 0^\circ$ (V), $Z_g = Z_0 = 50$ (Ω), $R_L = 25$ (Ω)
 $\Gamma = 0.52$

$$V_i = \frac{Z_i}{Z_0 + Z_i} V_g \quad I_i = \frac{V_g}{Z_0 + Z_i}$$

$$\text{where } Z_i = Z_0 \frac{0.5Z_0 + jZ_0 \tan \beta l}{Z_0 + j0.5Z_0 \tan \beta l} = Z_0 \frac{1+j2 \tan \beta l}{2+j \tan \beta l}$$

$$\therefore V_i = \frac{1+j2 \tan \beta l}{3(1+j \tan \beta l)} V_g = \frac{1}{30} \left(\frac{1+j2 \tan \beta l}{1+j \tan \beta l} \right) \text{ (V)}$$

$$I_i = \frac{2+j \tan \beta l}{3Z_0(1+j \tan \beta l)} V_g = \frac{2}{3} \left(\frac{2+j \tan \beta l}{1+j \tan \beta l} \right) \text{ (mA)}$$

Setting $Z_g = Z_0$ and $\Gamma_g = 0$ in Eqs. (9-120a) and (9-120b),

$$\begin{aligned} \text{we have } V_L = V(z=0) &= \frac{V_g Z_0}{Z_0 + Z_g} e^{-j\beta l} (1+\Gamma) \quad (\Gamma = \frac{R_L - Z_0}{R_L + Z_0}) \\ &= \frac{1}{30} e^{-j\beta l} \text{ (V)} \end{aligned}$$

$$I_L = I(z=0) = \frac{V_g}{Z_0 + Z_g} e^{-j\beta l} (1-\Gamma) = \frac{4}{3} e^{-j\beta l} \text{ (mA)}$$

b) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$

c) $(P_{av})_L = \frac{1}{2} \operatorname{Re}(V_L I_L^*) = \frac{1}{2} \left(\frac{1}{30} \right) \left(\frac{4}{3} \times 10^{-3} \right) = 2.22 \times 10^{-5} \text{ (W)}$
 $= 0.0222 \text{ (mW)}$

$$\text{If } R_L = 50 \text{ } (\Omega), \quad V_L = \frac{V_g}{2} e^{-j\beta l}, \quad I_L = \frac{V_g}{2Z_0} e^{-j\beta l}$$

$$\longrightarrow \text{Max. } (P_{av})_L = \frac{V_g^2}{8Z_0} = 2.50 \times 10^{-5} \text{ (W)}$$

P.9-27 $\Gamma_g = 0, \quad \Gamma = \frac{Z_L - R_0}{Z_L + R_0} = \frac{j-1}{j+1} = j, \quad z = l - z', \quad \beta = \omega \sqrt{LC}$

$$l = \lambda/4, \quad e^{-j\beta z} = e^{-j\beta(\lambda/4 - z')} = e^{-j(\pi/4 - \beta z')} = -j e^{j\beta z'}$$

From Eqs. (9-120a) and (9-120b):

a) $V(z) = -j \frac{V_g}{2} e^{j\beta z'} (1+j e^{j2\beta z'}) = 55 (e^{-j\beta z'} - j e^{j\beta z'}) \text{ (V)}$

$$I(z') = -j \frac{V_0}{2Z_0} e^{j\beta z'} (1 - j e^{-j2\beta z'}) = -1.1 (e^{j\beta z'} + j e^{j\beta z'}) \quad (A)$$

b) $v(z', t) = \text{Re} [V(z') e^{j\omega t}] = 55 [\sin(\omega t - \beta z') - \cos(\omega t + \beta z')] \quad (V)$
 $i(z', t) = -1.1 [\sin(\omega t - \beta z') + \cos(\omega t + \beta z')] \quad (A)$

c) At the load, $z' = 0$,

$$P_L(t) = v(0, t) i(0, t) = 60.5 (\cos^2 \omega t - \sin^2 \omega t) = 60.5 \cos(2\omega t) \quad (W)$$

$$V_L = \frac{V_0}{2} (1 - j), \quad I_L = -\frac{V_0}{2Z_0} (1 + j)$$

$$(P_{av})_L = \frac{1}{2} \text{Re} (V_L I_L^*) = \frac{V_0^2}{4Z_0} \text{Re} (2j) = 0$$

P.9-28 $f = 2 \times 10^8 \text{ (Hz)}, \quad \lambda = \frac{c}{f} = 1.5 \text{ (m)}$

a) Open-circuited line, $l = 1 \text{ (m)}, \quad l/\lambda = 0.667$.

Smith chart: Start from P_{oc} on the extreme right, rotate clockwise one complete revolution ($2\pi = \lambda/2$) and continue on for an additional 0.167λ to 0.417λ on the "wavelength toward generator" scale. Read $x = -j0.575 \rightarrow Z_i = 75 \times (-j0.575) = -j43.1 \text{ } (\Omega)$.

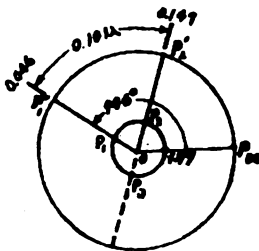
Draw a straight line from the $(0 - j0.575)$ point through the center and intersect at $(0 + j1.74)$ on the opposite side of the chart. $\rightarrow Y_i = \frac{1}{75} = (j1.74) = j0.0232 \text{ (S)}$.

b) Short-circuited line, $l = 0.8 \text{ (m)}, \quad l/\lambda = 0.533$.

Start from the extreme left point P_{sc} , rotate clockwise one complete revolution and continue on for an additional 0.033λ to read $x = j0.21 \rightarrow Z_i = 75 \times j0.21 = j15.8 \text{ } (\Omega)$.

Draw a straight line from the $(0 + j0.21)$ point through the center and intersect at $(0 - j4.75)$ on the opposite side of the chart. $\rightarrow Y_i = \frac{1}{75} = (-j4.75) = -j0.063 \text{ (S)}$.

P.9-29



$$z_L = \frac{1}{50} (30 + j10) = 0.6 + j0.2$$

- a) 1. Locate $z_L = 0.6 + j0.2$ on Smith chart (Point P_1).
 2. With center at O draw a 1r circle through P_1 , intersecting OP_{sc} at 1.77. $\rightarrow S = 1.77$.

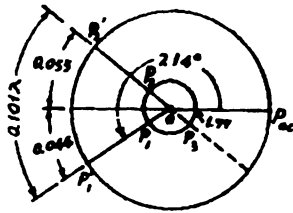
$$b) \Gamma = \frac{1.77-1}{1.77+1} e^{j146^\circ} = 0.28 e^{j146^\circ}$$

- c) 1. Draw line OP_1 , intersecting the periphery at P_1' .
 Read 0.046 on "wavelengths toward generator" scale.
 2. Move clockwise by 0.101λ to 0.147 (Point P_2').
 3. Join O and P_2' , intersecting the $|\Gamma|$ -circle at P_2 .
 4. Read $z_2 = 1 + j0.59$ at P_2 .
 $Z_2 = 50 z_2 = 50 + j29.5 (\Omega)$.

- d) Extend line $P_2'P_2O$ to P_3 . Read $y_3 = 0.75 - j0.43$.
 $Y_3 = \frac{1}{50} y_3 = 0.015 - j0.009 (S)$.

e) There is no voltage minimum on the line, but $V_L < V_i$.

P.9-30



$$z_L = \frac{1}{50} (30 - j10) = 0.6 - j0.2$$

- a) Locate $z_L = 0.6 - j0.2$ on Smith chart (Point P_1). With center at O draw a $|\Gamma|$ -circle through P_1 , intersecting line OP_{sc} at 1.77. $\rightarrow S = 1.77$.

$$b) \Gamma = 0.28 e^{j214^\circ}$$

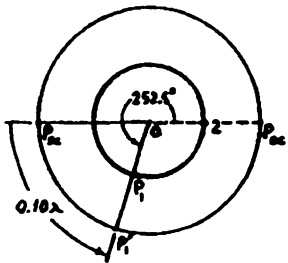
- c) 1. Draw line OP_1 , intersecting the periphery at P_1' .
 Read 0.454 on "wavelengths toward generator" scale.
 2. Move clockwise by 0.101λ to 0.055 (Point P_2').
 3. Join O and P_2' , intersecting the $|\Gamma|$ -circle at P_2 .
 4. Read $z_2 = 0.61 + j0.23$ at P_2 .
 $Z_2 = 50 z_2 = 30.5 + j11.5 (\Omega)$.

- d) Extend line $P_2'P_2O$ to P_3 . Read $y_3 = 1.42 - j0.54$.
 $Y_3 = \frac{1}{50} y_3 = 0.0284 - j0.0108 (S)$.

e) There is a voltage minimum at $z_m' = 0.046\lambda$.

P.9-31 $\lambda/2 = 25, \lambda = 50 (cm)$

First voltage minimum occurs at $z_m' = \frac{5}{50} = 0.1\lambda$.



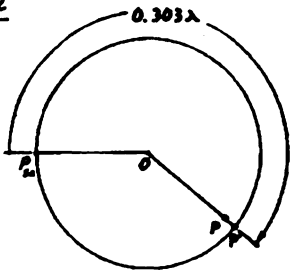
- a) 1. Start from P_{sc} and rotate counterclockwise 0.10λ toward the load to P_1' .
2. Draw the $|\Gamma|$ -circle, intersecting line OP_{oc} at 2 ($S=2$).
3. Join OP_1' , intersecting the $|\Gamma|$ -circle at P_1 .
4. Read $z_L = 0.675 - j0.475$.

$$Z_L = 50 z_L = 33.75 - j23.75 \text{ } (\Omega)$$

$$b) \Gamma = \frac{2-1}{2+1} e^{j\theta} = \frac{1}{3} e^{j252.5^\circ}$$

- c) If $Z_L = 0$, the first voltage minimum would be at $z_m' = \lambda/2 = 25 \text{ (cm)}$ from the short-circuit.

P. 9-32

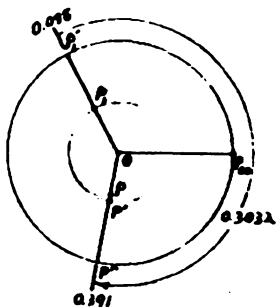


$$a) z_i = \frac{1}{100} (40 - j280) = 0.40 - j2.80$$

1. Enter z_i on Smith chart (Point P).
2. Join O and P, and extend to P' .
3. Read on "wavelengths toward generator" scale: 0.303.

$$\beta L = 0.606\pi, \quad L = 1.5 \text{ (m)} \longrightarrow \beta = 1.269 \text{ (rad/m)}$$

$$\frac{\overline{OP}}{OP'} = 0.915 \longrightarrow \alpha = \frac{1}{2L} \ln \frac{1}{0.915} = 0.0297 \text{ (Np/m)}$$



- b) 1. Enter $z_L = 0.5 + j0.5$ on Smith chart (Point P_1).
2. Draw line from O through P_1 to P_1' . Read on "wavelengths toward generator" scale: 0.088.
3. Move clockwise by 0.303λ to 0.391 (Point P'').

4. Join OP' , intersecting the $|Γ|$ -circle through P_1 at P' .

5. Mark point P on line OP' such that $\frac{OP}{OP'} = 0.915$.

6. Read at P : $z_1 = 0.625 - j0.59 \rightarrow Z_1 = 62.5 - j59.0 (\Omega)$

c) 1. Move clockwise from P_{22} on "wavelengths toward generator" scale to 0.15, say P' .

2. Join OP' .

3. Mark point P on line OP' such that

$$\frac{OP}{OP'} = e^{-2\alpha l} \frac{OP'}{OP'} = 0.957 \frac{OP'}{OP'}$$

4. Read at P : $z_1 = 0.065 + j1.38 \rightarrow Z_1 = 6.5 + j138 (\Omega)$.

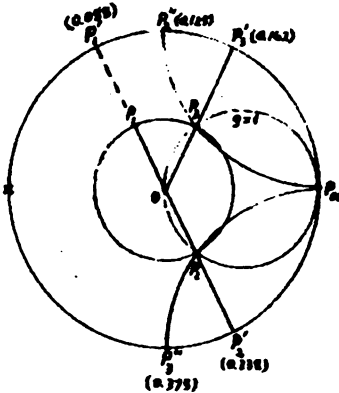
P.9-33 $f = 2 \times 10^8$ (Hz), $\lambda = 1.5$ (m) $\rightarrow l = \frac{\lambda}{4} = 0.375$ (m).

$$Z_0 = 73 \times 100 = 148 (\Omega)$$

For two-wire transmission line: $Z_0 = 120 \cosh^{-1} \left(\frac{D}{2a} \right)$.

$$D = 2$$
 (cm) $\rightarrow a = 0.54$ (cm).

P.9-34



$$z_1 = 0.5 + j0.5$$

$$y_1 = 1 - j$$

a) See construction.

$$P_1: z_2 = 0.5 + j0.5$$

$$P_2: y_2 = 1 - j = y_3 \rightarrow d_3 = 0$$

$$P_3: y_3 = 1 + j1$$

$$\rightarrow d_3 = 0.162\lambda + (0.5 - 0.115)\lambda = 0.324\lambda$$

$$P_2^*: b_2 = j1 \rightarrow d_2 = (0.5 + 0.115)\lambda = 0.375\lambda$$

$$P_3^*: b_3 = -j1 \rightarrow d_3 = (0.375 - 0.35)\lambda = 0.125\lambda$$

b) For $Z'_0 = 75 = 1.5 Z_0$, $Y'_0 = 0.667 Y_0$.

The required normalized stub admittances are $b'_2 = -b'_3 = \frac{j}{0.667} = j1$.

	$(Z_0)_{Stub} = (Z_0)_{Line}$	$(Z_0)_{Stub} = 1.5(Z_0)_{Line}$
$z_1 = 0.5 + j0.5$	$d_2 = 0, \quad \ell_2 = 0.375\lambda$	$d'_2 = 0, \quad \ell'_2 = 0.406\lambda$
$y_1 = 1 - j1$	$d_3 = 0.324\lambda, \quad \ell_3 = 0.125\lambda$	$d'_3 = 0.324\lambda, \quad \ell'_3 = 0.092\lambda$

P. 9-35 $z_L = 0.5 + j0.5$

Use Smith chart as an impedance chart. Same construction as that in problem P. 9-34 except P_{sc} would be on the extreme left (marked by a ∞), and $g=1$ circle becomes $r=1$ circle.

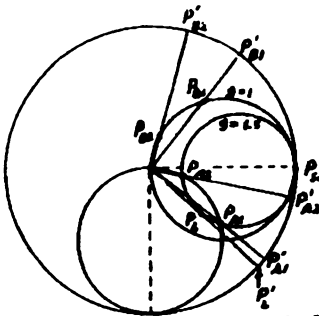
$P_1: z_L = 0.5 + j0.5; P_2: z_{in} = 1 + j1$ with $d_2 = (0.162 - 0.088)\lambda = 0.074\lambda$.

$P_3: z_{in} = 1 - j1$ with $d_3 = (0.338 - 0.088)\lambda = 0.250\lambda$.

To achieve a match with a series stub having $R'_0 = \frac{15}{50} R_0$, we need a normalized stub susceptance $-j \frac{50}{35} = -j1.43$ for solution corresponding to P_2 . From Smith chart we obtain the required stub length $l_2 = 0.347\lambda$.

Similarly for solution corresponding to P_3 , a stub with a normalized susceptance $+j1.43$ is needed, which requires a stub length $l_3 = 0.153\lambda$.

P. 9-36



$z_L = 0.33 + j0.33$

$P_L: y_L = 1.50 - j1.50$ (0.306λ at P'_L)

$P_{A1}: y_{A1} = 1.50 - j1.80$ (0.304λ at P'_{A1})

$P_{A2}: y_{A2} = 1.50 - j0.14$ (0.269λ at P'_{A2})

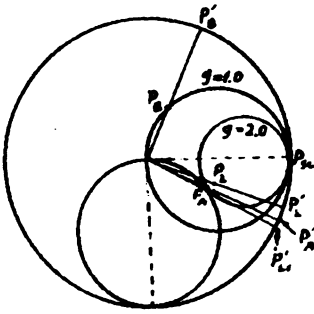
$P_{B1}: y_{B1} = 1.00 + j1.60$ (0.179λ at P'_{B1})

$P_{B2}: y_{B2} = 1.00 + j0.40$ (0.144λ at P'_{B2})

a) Short-circuited stubs b) Open-circuited stubs

$(y_{sc})_1 = y_{A1} - y_L = -j0.30$	$l_{A1} = 0.203\lambda$	$l_{A1} = 0.453\lambda$
$(y_{sc})_2 = y_{A2} - y_L = j1.36$	$l_{A2} = 0.399\lambda$	$l_{A2} = 0.149\lambda$
$(y_{sc})_1 = -j1.60$	$l_{B1} = 0.089\lambda$	$l_{B1} = 0.339\lambda$
$(y_{sc})_2 = -j0.40$	$l_{B2} = 0.189\lambda$	$l_{B2} = 0.439\lambda$

P. 9-37



$$y_L = \frac{100}{100 + j50} = 2.4 - j1.2$$

Point P_L on Smith chart.
(0.280λ at P'_L)

Since the rotated $g=1.0$ circle is tangent to the $g=2.0$ circle, an added line length d_L is needed to convert $g_L(2.4)$ to 2.0,

moving from P_L along the $|\Gamma|$ -circle to P_{L1} (not shown on the $g=2.0$ circle (0.291λ at P'_{L1}). Note that P_{L1} is different from P_A , the point of tangency between the $g=2.0$ and rotated $g=1.0$ circles.

a) $\text{Min. } d_L = 0.291\lambda - 0.280\lambda = 0.011\lambda.$

b) $P_A : y_A = 2 - j1$ (0.287λ at P'_A).

$P_B : y_B = 1 + j1$ (0.162λ at P'_B).

$Y_{SA} = Y_A - Y_{L1} = (2 - j1) - (2 - j1.35) = j0.35 \rightarrow \ell_A = 0.304\lambda$

$Y_{SB} = -j1 \rightarrow \ell_B = 0.125\lambda.$

P. 9-38 Let $\theta = \beta d_o = \frac{2\pi}{\lambda} d_o.$

Require: $g_L \leq \frac{1}{\sin^2 \theta}$ (Analytic solution!)

d_o	θ	g_L
$\lambda/16$	22.5°	≤ 6.83
$\lambda/8$	45°	≤ 2.0
$\lambda/4$	90°	≤ 1.0
$3\lambda/8$	135°	≤ 2.0
$7\lambda/16$	157.5°	≤ 6.83

[†] See D.K. Cheng and C.H. Liang, "Computer Solution of Double-Stub Impedance-Matching Problems," IEEE Transactions on Education, vol. E-25, pp. 120-123, November 1982.

Chapter 10

10-1

From $\nabla \times \bar{E} = -j\omega\mu\bar{H}$

$$\frac{\partial E_z^0}{r\partial\phi} + \gamma E_\phi^0 = j\omega\mu H_r^0$$

$$-\gamma E_r^0 - \frac{\partial E_z^0}{\partial r} = -j\omega\mu H_\phi^0$$

$$\frac{\partial(rE_\phi^0)}{r\partial r} - \frac{\partial E_r^0}{r\partial\phi} = -j\omega\mu H_z^0$$

From $\nabla \times \bar{H} = j\omega\epsilon\bar{E}$

$$\frac{\partial H_z^0}{r\partial\phi} + \gamma H_\phi^0 = j\omega\epsilon E_r^0$$

$$-\gamma H_r^0 - \frac{\partial H_z^0}{\partial r} = j\omega\epsilon E_\phi^0$$

$$\frac{\partial(rH_\phi^0)}{r\partial r} - \frac{\partial H_r^0}{r\partial\phi} = j\omega\epsilon E_z^0$$

Combining:

$$E_r^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z^0}{\partial r} + j\omega\mu \frac{\partial H_z^0}{r\partial\phi} \right)$$

$$E_\phi^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z^0}{r\partial\phi} - j\omega\mu \frac{\partial H_z^0}{\partial r} \right)$$

$$H_r^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z^0}{\partial r} - j\omega\epsilon \frac{\partial E_z^0}{r\partial\phi} \right)$$

$$H_\phi^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z^0}{r\partial\phi} + j\omega\epsilon \frac{\partial E_z^0}{\partial r} \right), \quad h^2 = \gamma^2 + \omega^2\mu\epsilon.$$

$E_z = E_z^0 e^{-\gamma z}$, where E_z^0 satisfies the following homogeneous

Helmholtz's equation: $\nabla_{\phi r}^2 E_z^0 + h^2 E_z^0 = 0$, or

$$\frac{\partial^2 E_z^0}{\partial r^2} + \frac{1}{r^2} \frac{\partial E_z^0}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z^0}{\partial \phi^2} + h^2 E_z^0 = 0.$$

Similarly for $H_z = H_z^0 e^{-\gamma z}$.

10-2

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad (1)$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \quad (2)$$

From (1), $(\bar{\nabla}_T + \bar{a}_z \frac{\partial}{\partial z}) \times (\bar{E}_T + \bar{a}_z E_z) = -j\omega\mu(\bar{H}_T + \bar{a}_z H_z)$

$$\longrightarrow \bar{\nabla}_T \times (\bar{a}_z E_z) + \gamma \bar{E}_T \times \bar{a}_z = -j\omega\mu \bar{H}_T$$

$$\longrightarrow \bar{\nabla}_T E_z \times \bar{a}_z + \gamma \bar{E}_T \times \bar{a}_z = -j\omega\mu \bar{H}_T \quad (3)$$

$$(\because \bar{\nabla}_T \times (\bar{a}_z E_z) = \bar{\nabla}_T E_z \times \bar{a}_z)$$

Similarly from (2) we obtain

$$\bar{\nabla}_T H_z \times \bar{a}_z + \gamma \bar{H}_T \times \bar{a}_z = j\omega\epsilon \bar{E}_T \quad (4)$$

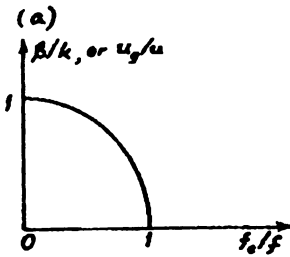
Combining (3) and (4), we have

$$\omega^2\mu\epsilon \bar{E}_T = \gamma [\bar{\nabla}_T E_z \times \bar{a}_z + \gamma \bar{E}_T \times \bar{a}_z] \times \bar{a}_z - j\omega\mu \bar{\nabla}_T H_z \times \bar{a}_z$$

$$\longrightarrow \bar{E}_T = -\frac{1}{h^2} (\gamma \bar{\nabla}_T E_z - \bar{a}_z j\omega\mu \times \bar{\nabla}_T H_z), \quad h^2 = \gamma^2 + \omega^2\mu\epsilon$$

Similarly, $\bar{H}_T = -\frac{1}{h^2} (\gamma \bar{\nabla}_T H_z + \bar{a}_z j\omega\epsilon \times \bar{\nabla}_T E_z)$.

P.10-3



From Eq. (10-33):

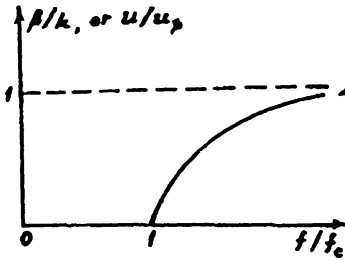
$$\left(\frac{\beta}{k}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$

From Eq. (10-37):

$$\left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$

Both are equations of a unit circle.

b)



From Eq. (10-33):

$$\left(\frac{\beta}{k}\right)^2 = 1 - \frac{1}{(f/f_c)^2}$$

From Eq. (10-36):

$$\left(\frac{u}{u_p}\right)^2 = 1 - \frac{1}{(f/f_c)^2}$$

From Eq. (10-34):

$$\left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{(f/f_c)^2}{(f/f_c)^2 - 1}$$

c) At $f/f_c = 1.25$:

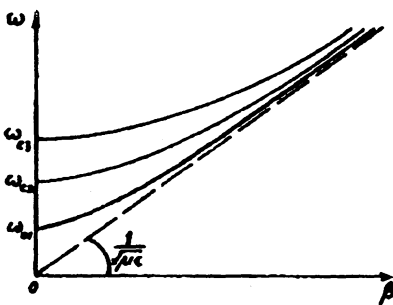
$$u_p/u = 1.67$$

$$u_g/u = 0.60$$

$$\beta/k = 0.60$$

$$\lambda_g/\lambda = 1.67$$

P.10-4 a) For parallel-plate waveguides:



$$\omega^2 \mu \epsilon - \beta^2 = \left(\frac{n\pi}{b}\right)^2$$

$$\omega_{c1} = \frac{\pi}{b\sqrt{\mu\epsilon}}$$

$$\omega_{c2} = \frac{2\pi}{b\sqrt{\mu\epsilon}}$$

$$\omega_{c3} = \frac{1}{b\sqrt{\mu\epsilon}}$$

a) Constitutive parameters ϵ and μ affect both ω_c and the slope of the ω - β curves; b affects ω_c

but not the slope at high-frequencies. b) Yes.

P.10-5 Field expressions for TM_n modes, from Eqs. (10-54 a, b, & c):

$$E_x^0(y) = A_n \sin(n\pi y/b)$$

$$H_x^0(y) = \frac{j\omega\epsilon}{h} A_n \cos(n\pi y/b)$$

$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos(n\pi y/b).$$

Surface charge densities:

$$\rho_{sL} = \bar{a}_n \cdot \bar{D} \Big|_{y=0} = \epsilon E_y^0(0) = -\frac{\gamma\epsilon}{h} A_n$$

$$\rho_{sU} = \bar{a}_n \cdot \bar{D} \Big|_{y=b} = -\epsilon E^0(b) = (-1)^n \frac{\gamma\epsilon}{h} A_n$$

Surface current densities:

$$\bar{J}_{sL} = \bar{a}_n \times \bar{H} \Big|_{y=0} = \bar{a}_y \times \bar{H}(0) = -\bar{a}_z \frac{j\omega\epsilon}{h} A_n$$

$$\bar{J}_{sU} = \bar{a}_n \times \bar{H} \Big|_{y=b} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_z (-1)^n \frac{j\omega\epsilon}{h} A_n = \begin{cases} \bar{J}_{sL} & \text{for } n \text{ odd} \\ -\bar{J}_{sL} & \text{for } n \text{ even.} \end{cases}$$

P.10-6 Field expressions for TE_n modes, from Eqs. (10-68 a, b, & c):

$$H_x^0(y) = B_n \cos(n\pi y/b)$$

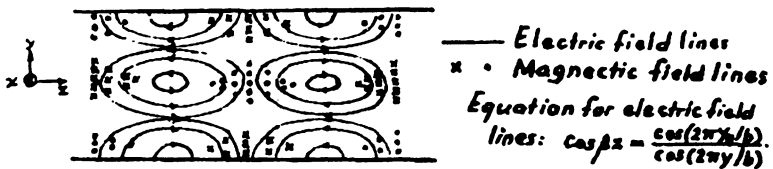
$$H_y^0(y) = \frac{\gamma}{h} B_n \sin(n\pi y/b)$$

$$E_x^0(y) = \frac{j\omega\mu}{h} B_n \sin(n\pi y/b)$$

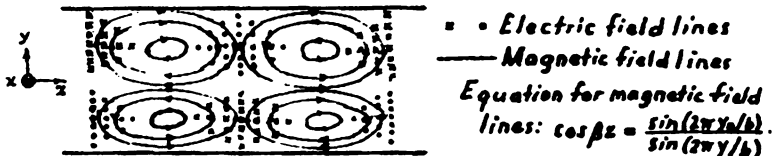
$$\bar{J}_{sL} = \bar{a}_y \times \bar{H}(0) = \bar{a}_x B_n$$

$$\bar{J}_{sU} = -\bar{a}_y \times \bar{H}(b) = \bar{a}_x (-1)^n B_n = \begin{cases} \bar{J}_{sL} & \text{for } n \text{ odd} \\ -\bar{J}_{sL} & \text{for } n \text{ even.} \end{cases}$$

P.10-7 a) Set $n=2$ in the field expressions in problem P.10-5.



b) Set $n=2$ in the field expressions in problem P.10-6.



P.10-8 Given: $\sigma_c = 5.80 \times 10^7$ (S/m), $\epsilon_r = 2.25$, $\mu_r = 1$
 $\sigma = 10^{-10}$ (S/m), $b = 5 \times 10^{-2}$ (m), $f = 10^{10}$ (Hz)

a) TEM mode

$$\beta = \omega \sqrt{\mu \epsilon} = 314.2 \text{ (rad/m)}$$

$$\alpha_d = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} = 1.257 \times 10^{-8} \text{ (Np/m)}$$

$$\alpha_c = \frac{1}{b} \sqrt{\frac{\pi f \epsilon}{\sigma_c}} = 2.076 \times 10^{-3} \text{ (Np/m)}$$

$$u_p = u_g = u = \frac{1}{\sqrt{\mu \epsilon}} = 2 \times 10^8 \text{ (m/s)}$$

$$\lambda_g = \lambda = \frac{u}{f} = 2 \times 10^{-2} \text{ (m)}$$

b) TM₁ mode — $(f_c)_{TM_1} = \frac{1}{2b\sqrt{\mu \epsilon}} = 2 \times 10^9 \text{ (Hz)} < f$.

$$F_1 = \sqrt{1 - (f_c/f)^2} = 0.9798.$$

$$\beta = \omega \sqrt{\mu \epsilon} \cdot F_1 = 307.8 \text{ (rad/m)}$$

$$\alpha_d = \frac{\sigma \eta}{2F_1} = 1.283 \times 10^{-8} \text{ (Np/m)}$$

$$\alpha_c = \frac{2R_s}{\eta b F_1} = \frac{2}{b F_1} \sqrt{\frac{\pi f \epsilon}{\sigma_c}} = 4.238 \times 10^{-3} \text{ (Np/m)}$$

$$u_p = u/F_1 = 2.041 \times 10^8 \text{ (m/s)}$$

$$u_g = u \cdot F_1 = 1.960 \times 10^8 \text{ (m/s)}$$

$$\lambda_g = \lambda/F_1 = 2.041 \times 10^{-2} \text{ (m)}$$

c) TM₂ mode — $(f_c)_{TM_2} = \frac{1}{b\sqrt{\mu \epsilon}} = 4 \times 10^9 \text{ (Hz)} < f$.

$$F_2 = \sqrt{1 - (f_c/f)^2} = 0.9165.$$

$$\beta = \omega \sqrt{\mu \epsilon} \cdot F_2 = 287.9 \text{ (rad/m)}$$

$$\alpha_d = \frac{\sigma \eta}{2F_2} = 1.371 \times 10^{-8} \text{ (Np/m)}$$

$$\alpha_c = \frac{2}{b F_2} \sqrt{\frac{\pi f \epsilon}{\sigma_c}} = 4.530 \times 10^{-3} \text{ (Np/m)}$$

$$u_p = u/F_2 = 2.182 \times 10^8 \text{ (m/s)}$$

$$u_g = u \cdot F_2 = 1.833 \times 10^8 \text{ (m/s)}$$

$$\lambda_g = \lambda/F_2 = 2.182 \times 10^{-2} \text{ (m)}$$

P.10-9 a) TE₁ mode — $(f_c)_{TE_1} = (f_c)_{TM_1} = 2 \times 10^9 \text{ (Hz)} < f$.

All required quantities are the same as those for the TM₁ mode in problem P.10-8 (b), except α_c . Using

Eq. (10-83), we have

$$\alpha_c = \frac{2}{b F_1} \sqrt{\frac{\pi f \epsilon}{\sigma_c}} \left(\frac{f_c}{f} \right)^2 = 1.695 \times 10^{-4} \text{ (Np/m)}$$

b) TE_2 mode ——— $(f_c)_{TE_2} = (f_c)_{TM_2} = 4 \times 10^9$ (Hz) $< f$.

All required quantities are the same as those for the TM_2 mode in problem P.10-8 (c), except d_c .

$$d_c = \frac{2}{bF_2} \sqrt{\frac{\pi \mu_0 \epsilon_0}{\epsilon_c}} \left(\frac{f_c}{f} \right)^2 = 7.249 \times 10^{-9} \text{ (Np/m)}.$$

P.10-10 For TM_n modes in a parallel-plate waveguide,

$$d_c = \frac{2}{\eta b} \sqrt{\frac{\pi \mu_0 f_c}{\epsilon_c}} \frac{1}{\sqrt{(f_c/f)[1-(f_c/f)^2]}}$$

$$= \frac{2}{\eta b} \sqrt{\frac{\pi \mu_0 f_c}{\epsilon_c}} \frac{1}{\sqrt{F(x)}}$$

where $F(x) = 1 - x^2$, $x = f_c/f$.

a) To find minimum d_c , set

$$\frac{\partial F(x)}{\partial x} = 1 - 3x^2 = 0 \longrightarrow x = \frac{1}{\sqrt{3}}$$

$$\therefore f = \sqrt{3} f_c.$$

b) At $f_c/f = 1/\sqrt{3}$, $\frac{1}{\sqrt{F(x)}} = 1.612$,

and $\min. d_c = \frac{3.224}{\eta b} \sqrt{\frac{\pi \mu_0 b}{\epsilon_c}}$.

c) For $\epsilon_c = 5.80 \times 10^7$ (S/m), $b = 5 \times 10^{-2}$ (m), $\eta = 120\pi$ (Ω),
and $\mu_0 = 4\pi \times 10^{-7}$ (H/m),

$$(f_c)_{TM_2} = \frac{1}{2b/\mu_0 \epsilon_c} = 3 \times 10^9 \text{ (Hz)}$$

$$\min. d_c = 2.444 \times 10^{-7} \text{ (Np/m)}.$$

P.10-11 Parallel-plate waveguide: $b = 0.03$ (m), $f = 10^{10}$ (Hz).

a) TEM mode

From Eqs. (9-1a) and (9-1b):

$$\begin{cases} E_y^o = E_0 \\ H_x^o = -\frac{E_0}{\eta_0} \end{cases}$$

$$P_{av} = \frac{w}{2} \int_0^b -E_y^o H_x^o dy = \frac{wb}{2\eta_0} E_0^2.$$

Dielectric strength of air: Max. $E_0 = 3 \times 10^6$ (V/m).

$$\text{Max.} \left(\frac{P_{av}}{w} \right) = \frac{b}{2\eta_0} (3 \times 10^6)^2 = 358 \times 10^8 \text{ (W/m)} = 358 \text{ (MW/m)}.$$

b) TM₁ mode

From Eqs. (10-54b) and (10-54c):

$$\begin{cases} E_y^0(y) = E_0 \cos\left(\frac{\pi y}{b}\right) \\ H_x^0(y) = -\frac{E_0}{\eta_0 \sqrt{1-(f_c/f)^2}} \cos\left(\frac{\pi y}{b}\right) \end{cases}$$

$$f_c = \frac{1}{2b/\mu_0 \epsilon_0} = 5 \times 10^9 \text{ (Hz)}$$

$$P_{av} = \frac{w}{2} \int_0^b -E_y^0(y) H_x^0(y) dy = \frac{wb E_0^2}{4\eta_0 \sqrt{1-(f_c/f)^2}}$$

$$\text{Max.} \left(\frac{P_{av}}{w}\right) = \frac{b(3 \times 10^6)^2}{4\eta_0 \sqrt{1-(f_c/f)^2}} = 2.07 \times 10^8 \text{ (W/m)} = 207 \text{ (MW/m)}$$

c) TE₁ mode

From Eqs. (10-68b) and (10-68c):

$$\begin{cases} E_x^0(y) = E_0 \sin\left(\frac{\pi y}{b}\right) \\ H_y^0(y) = \frac{E_0}{\eta_0} \sqrt{1-(f_c/f)^2} \sin\left(\frac{\pi y}{b}\right) \end{cases}$$

$$P_{av} = \frac{w}{2} \int_0^b E_x^0(y) H_y^0(y) dy = \frac{wb E_0^2}{4\eta_0} \sqrt{1-(f_c/f)^2}$$

$$\text{Max.} \left(\frac{P_{av}}{w}\right) = \frac{b(3 \times 10^6)^2}{4\eta_0} \sqrt{1-(f_c/f)^2} = 1.55 \times 10^8 \text{ (W/m)} = 155 \text{ (MW/m)}$$

P.10-12 $f = \frac{u}{\lambda}, \quad f_c = \frac{u}{\lambda_c}$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

$$\longrightarrow \frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

P.10-13 Equations (10-94a) through (10-94d) for TM₁₁ mode:

$$E_x^0(x,y) = \frac{-j\beta_0}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$E_y^0(x,y) = \frac{-j\beta_0}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x,y) = E_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

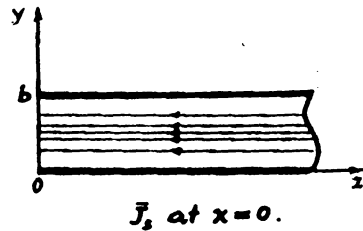
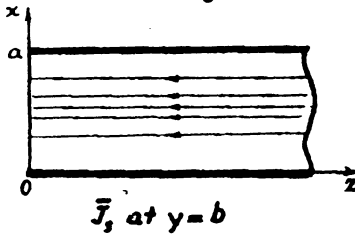
$$H_x^0(x,y) = \frac{j\omega\epsilon}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

$$H_y^0(x,y) = \frac{-j\omega\epsilon}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

a) Surface current densities:

$$\begin{aligned}\bar{J}_s (y=0) &= \bar{a}_n \times \bar{H} \Big|_{y=0} = \bar{a}_y \times [\bar{a}_x H_x^o(x,0) + \bar{a}_y H_y^o(x,0)] \\ &= -\bar{a}_z H_x^o(x,0) = -\bar{a}_z \frac{j\omega\epsilon}{h^1} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) e^{j\beta z} \\ &= \bar{J}_s (y=b).\end{aligned}$$

$$\begin{aligned}\bar{J}_s (x=0) &= \bar{a}_n \times \bar{H} \Big|_{x=0} = \bar{a}_x \times [\bar{a}_x H_x^o(0,y) + \bar{a}_y H_y^o(0,y)] \\ &= \bar{a}_z H_y^o(0,y) = -\bar{a}_z \frac{j\omega\epsilon}{h^1} \left(\frac{\pi}{a}\right) E_0 \sin\left(\frac{\pi y}{b}\right) e^{j\beta z} \\ &= \bar{J}_s (x=a).\end{aligned}$$



10-14 $(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1}{2a\sqrt{\mu\epsilon}} F(m,n).$

a) $a=2b, F(m,n) = \sqrt{m^2 + 4n^2}$

Modes	$F(m,n)$
TE_{10}	1
TE_{01}, TE_{20}	2
TE_{11}, TM_{11}	$\sqrt{5}$
TE_{02}	4
TM_{12}	$\sqrt{17}$
TM_{22}	$\sqrt{20}$

b) $a=b, F(m,n) = \sqrt{m^2 + n^2}$.

Modes	$F(m,n)$
TE_{10}, TE_{01}	1
TE_{11}, TM_{11}	$\sqrt{2}$
TE_{02}, TE_{20}	2
TM_{12}	$\sqrt{5}$
TM_{22}	$2\sqrt{2}$

10-15 $f = 3 \times 10^9$ (Hz), $\lambda = c/f = 0.1$ (m).

Let $a = kb, 1 < k < 2$. $(f_c)_{mn} = \frac{3 \times 10^9}{2a} \sqrt{m^2 + k^2 n^2}$.

a) $(f_c)_{10} = \frac{1.5 \times 10^9}{a}$ for the dominant TE_{10} mode.

For $f > 1.2 (f_c)_{10}$: $a > 0.06$ (m).

The next higher-order mode is TE_{01} with $(f_c)_{01} = \frac{1.5 \times 10^9}{b}$.

For $f < 0.8 (f_c)_{01}$: $b < 0.04$ (m).

We choose $a = 6.5$ (cm) and $b = 3.5$ (cm).

$$b) \quad u_p = \frac{c}{\sqrt{1-(\lambda/2a)^2}} = 4.70 \times 10^8 \text{ (m/s)}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/2a)^2}} = 0.157 \text{ (m)} = 15.7 \text{ (cm)}$$

$$\beta = \frac{2\pi}{\lambda_g} = 40.1 \text{ (rad/m)}$$

$$(Z_{TE})_{10} = \frac{\eta_0}{\sqrt{1-(\lambda/2a)^2}} = 590 \text{ (\Omega)}$$

P.10-16 Given: $a = 2.5 \times 10^{-2} \text{ (m)}$, $b = 1.5 \times 10^{-2} \text{ (m)}$, $f = 7.5 \times 10^8 \text{ (Hz)}$

$$a) \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{7.5 \times 10^8} = 0.04 \text{ (m)}$$

$$F_1 = \sqrt{1-(\lambda/2a)^2} = 0.60$$

$$\lambda_g = \lambda/F_1 = 0.0667 \text{ (m)} = 6.67 \text{ (cm)}$$

$$\beta = 2\pi/\lambda_g = 94.2 \text{ (rad/m)}$$

$$u_p = c/F_1 = 5 \times 10^8 \text{ (m/s)}$$

$$u_g = c \cdot F_1 = 1.8 \times 10^8 \text{ (m/s)}$$

$$(Z_{TE})_{10} = \eta_0/F_1 = 200\pi = 628 \text{ (\Omega)}$$

$$b) \quad \lambda' = \frac{u}{f} = \frac{\lambda}{\sqrt{2}} = 0.0283 \text{ (m)}$$

$$F_2 = \sqrt{1-(\lambda'/2a)^2} = 0.825$$

$$\lambda'_g = \lambda'/F_2 = 0.0343 \text{ (m)} = 3.43 \text{ (cm)}$$

$$\beta' = 2\pi/\lambda'_g = 183.2 \text{ (rad/m)}$$

$$u'_p = u/F_2 = 2.57 \times 10^8 \text{ (m/s)}$$

$$u'_g = u \cdot F_2 = 1.75 \times 10^8 \text{ (m/s)}$$

$$(Z_{TE})_{10} = \frac{\eta_0}{\sqrt{2} F_2} = 323 \text{ (\Omega)}$$

P.10-17 Given: $a = 7.20 \times 10^{-2} \text{ (m)}$, $b = 3.40 \times 10^{-2} \text{ (m)}$, $f = 3 \times 10^9 \text{ (Hz)}$

$$a) \quad \lambda_c = 2a = 14.40 \times 10^{-2} \text{ (m)}$$

$$f_c = \frac{c}{\lambda_c} = 2.08 \times 10^9 \text{ (Hz)}$$

$$b) \quad \lambda = \frac{c}{f} = 0.1 \text{ (m)}, \quad \sqrt{1-(\frac{\lambda}{2a})^2} = 0.720$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\frac{\lambda}{2a})^2}} = 0.139 \text{ (m)}$$

$$c) \quad R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 1.429 \times 10^{-2} \text{ (\Omega)} \longrightarrow (Z_c)_{TE_{10}} = \frac{R_s [1 + \frac{2b}{a} (\frac{f_c}{f})^2]}{\eta_0 b \sqrt{1-(f_c/f)^2}} = 2.26 \times 10^{-3} \text{ (N)}$$

$$d) \quad e^{-\alpha z} = \frac{1}{2} \longrightarrow z = \frac{1}{\alpha_c} \ln 2 = 307 \text{ (m)}$$

D. 10-18 Given: $a = 2.25 \times 10^{-2} \text{ (m)}$, $b = 1.00 \times 10^{-2} \text{ (m)}$, $f = 10^{10} \text{ (Hz)}$.

a) $\lambda = \frac{c}{f} = 3 \times 10^{-2} \text{ (m)}$, $\lambda_c = 2a = 4.50 \times 10^{-2} \text{ (m)}$

$$\sqrt{1 - (f_c/f)^2} = \sqrt{1 - (\lambda/\lambda_c)^2} = 0.745$$

$$\text{Eq. (10-119): } (\alpha_c)_{\eta_0} = \frac{1}{\eta_0 b} \sqrt{\frac{\pi f \mu_0}{\epsilon_c [1 - (f_c/f)^2]}} \left[1 + \frac{2b}{a} \left(\frac{f_c}{f} \right)^2 \right]$$

$$= 1.295 \times 10^{-2} \text{ (Np/m)}$$

b) From Eqs. (10-104a), (10-104b), and (10-103):

$$E_y^0 = E_0 \sin\left(\frac{\pi x}{a}\right)$$

$$H_x^0 = -\frac{E_0}{\eta_0} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \sin\left(\frac{\pi x}{a}\right)$$

$$H_z^0 = j \left(\frac{f_c}{f}\right) \frac{E_0}{\eta_0} \cos\left(\frac{\pi x}{a}\right)$$

$$P_{av} = \frac{1}{2} \int_0^b \int_0^a (-E_y^0 H_x^0) dx dy = \frac{E_0^2 ab}{4 \eta_0} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

For $P_{av} = 10^3 \text{ (W)}$ at the load (antenna), assuming under matched conditions:

$$|E_y^0| = E_0 = 94,800 \text{ (V/m)}, |H_x^0| = 187.4 \text{ (A/m)}, |H_z^0| = 167.6 \text{ (A/m)}$$

The waveguide is 1(m) long. \rightarrow The field intensities are higher at the sending end by a factor of $e^{2\alpha_c l} = 1.138$.

$$\therefore \text{Max. } |E_y^0| = 10,788 \text{ (V/m)}$$

$$\text{Max. } |H_x^0| = 213.3 \text{ (A/m)}$$

$$\text{Max. } |H_z^0| = 190.7 \text{ (A/m)}$$

$$c) \bar{J}(x=0) = \bar{a}_x \times (\bar{a}_x H_x^0 + \bar{a}_z H_z^0) \Big|_{x=0} = -\bar{a}_y H_x^0(0, y) = -\bar{a}_y j \left(\frac{f_c}{f}\right) \frac{E_0}{\eta_0}$$

$$|\bar{J}(x=0)| = |H_x^0| = 167.6 \text{ (A/m)}$$

$$\bar{J}(y=0) = \bar{a}_y \times (\bar{a}_x H_x^0 + \bar{a}_z H_z^0) \Big|_{y=0} = -\bar{a}_x H_x^0(x, 0) + \bar{a}_z H_z^0(x, 0)$$

$$|\bar{J}(y=0)| = [(H_x^0)^2 + (H_z^0)^2]^{1/2} = \frac{E_0}{\eta_0} \left\{ \left(\frac{f_c}{f}\right)^2 + \left[1 - 2\left(\frac{f_c}{f}\right)^2 \sin^2\left(\frac{\pi x}{a}\right)\right]^{1/2} \right\}$$

which is maximum at $x = a/2$.

At the sending end: $\text{Max. } |\bar{J}| = \frac{E_0}{\eta_0} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 1.138 = 213.3 \text{ (A/m)}$.

d) Total amount of average power dissipated in 1 (m) of waveguide:

$$P_d = 1000 (e^{2\alpha_c l} - 1) = 1000 (e^{0.0119} - 1) = 26.2 \text{ (W)}$$

P.10-19 From problem P.10-18, we have

$$P_{av} = \frac{E_0^2 ab}{4\eta_0} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 0.745$$

$$\therefore \text{Max. } P_{av} = \frac{(3 \times 10^6)^2 \times (2.25 \times 10^{-6})}{4 \times 120\pi} \times 0.745 = 10 \text{ (W)} \\ = 1 \text{ (MW)}$$

P.10-20 Let $A = \frac{1}{\eta_0 b} \sqrt{\frac{\pi f_c \mu_0}{\epsilon_c}}$ and $x = \frac{f_c}{f}$ in Eq. (10-119).

We write $(\alpha_c)_{TE_{10}} = AF(x)$, where $F(x) = \frac{1 + \frac{2b}{a}x^2}{\sqrt{x(1-x^2)}}$

For min. $(\alpha)_{TE_{10}}$, set $\frac{dF(x)}{dx} = 0$.

$$\longrightarrow x = \frac{f_c}{f} = \sqrt{\frac{3}{2}} \left[\left(1 + \frac{a}{2b}\right) - \sqrt{\left(1 + \frac{a}{2b}\right)^2 - \frac{2a}{9b}} \right]^{1/2}$$

P.10-21 Field expressions for TM_{11} mode from Eqs. (10-92) and (10-94):

$$E_x^0(x, y) = -\frac{j\beta_{11}}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$E_y^0(x, y) = -\frac{j\beta_{11}}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

$$E_z^0(x, y) = E_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$H_x^0(x, y) = \frac{j\omega\epsilon}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

$$H_y^0(x, y) = -\frac{j\omega\epsilon}{h^2} \left(\frac{\pi}{a}\right) E_0 \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

Calculate α_c from Eq. (10-77): $\alpha_c = \frac{P_L(z)}{2P(z)}$

$$P(z) = \frac{1}{2} \int_0^b \int_0^a [E_x^0 H_y^0 - E_y^0 H_x^0] dx dy = \frac{\omega\epsilon\beta_{11} E_0^2 ab}{8 \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right]}$$

From problem P.10-13:

$$\bar{J}_s(y=0) = \bar{J}_s(y=b) = -\bar{a}_z \frac{j\omega\epsilon}{h^2} \left(\frac{\pi}{b}\right) E_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{11}z}$$

$$\bar{J}_s(x=0) = \bar{J}_s(x=a) = -\bar{a}_z \frac{j\omega\epsilon}{h^2} \left(\frac{\pi}{a}\right) E_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{11}z}$$

$$P_L(z) = 2 [P_L(z)]_{x=0} + 2 [P_L(z)]_{y=0}$$

$$[P_L(z)]_{x=0} = \frac{1}{2} \int_0^b |\bar{J}_s(x=0)|^2 R_s dy = \frac{(\omega\epsilon)^2 R_s}{4h^4} \left(\frac{\pi}{a}\right)^2 E_0^2 b$$

$$[P_L(z)]_{y=0} = \frac{1}{2} \int_0^a |\bar{J}_s(y=0)|^2 R_s dx = \frac{(\omega\epsilon)^2 R_s}{4h^4} \left(\frac{\pi}{b}\right)^2 E_0^2 a$$

$$\rho_L(z) = \frac{(\omega\epsilon)^2 R_s E_p^2}{2 \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right]} \left[\left(\frac{\pi}{a}\right)^2 b + \left(\frac{\pi}{b}\right)^2 a \right]$$

$$\therefore (\alpha_c)_{TM_{11}} = \frac{2R_s (b/a^2 + a/b^2)}{\pi ab \sqrt{1 - (f_c/f)^2} (1/a^2 + 1/b^2)}$$

10-22 From Eqs. (10-124) and (10-126):

$$\text{Inside the slab: } \beta^2 = \omega^2 \mu_d \epsilon_d - k_y^2 < \omega^2 \mu_d \epsilon_d$$

$$\text{Outside the slab: } \beta^2 = \omega^2 \mu_0 \epsilon_0 + \alpha^2 > \omega^2 \mu_0 \epsilon_0$$

$$\therefore \omega \sqrt{\mu_0 \epsilon_0} < \beta < \omega \sqrt{\mu_d \epsilon_d}$$

$$\text{and } \frac{1}{\sqrt{\mu_0 \epsilon_0}} > u_T = \frac{\omega}{\beta} > \frac{1}{\sqrt{\mu_d \epsilon_d}}$$

10-23 From Eqs. (10-131a) and (10-130):

$$\left(\frac{ad}{2}\right)^2 + \left(\frac{k_y d}{2}\right)^2 = \left(\frac{k_0 d}{2}\right)^2 \left(\frac{\mu_r \epsilon_d}{\mu_0 \epsilon_0} - 1\right) \quad (1)$$

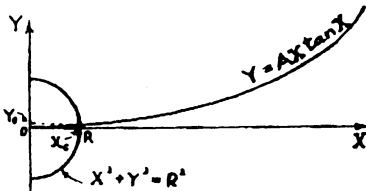
$$\frac{\epsilon_d}{\epsilon_0} \left(\frac{ad}{2}\right) = \left(\frac{k_y d}{2}\right) \tan\left(\frac{k_y d}{2}\right) \quad (2)$$

$$\text{Let } X = k_y d/2, Y = ad/2, A = \epsilon_0/\epsilon_d, \text{ and } R = \frac{k_0 d}{2} \sqrt{\frac{\mu_r \epsilon_d}{\mu_0 \epsilon_0} - 1}.$$

$$\text{Eqs. (1) and (2) become } \begin{cases} X^2 + Y^2 = R^2 & (3) \\ Y = AX \tan X & (4) \end{cases}$$

a) $f = 2 \times 10^8$ (Hz), $\lambda = c/f = 1.5$ (m).

$$k_0 d/2 = \pi d/\lambda = 0.0209, \quad A = \epsilon_0/\epsilon_d = 0.308, \quad R = 0.0314.$$



Graphical solution:

$$X_0 = 0.0314, \quad Y_0 = 3.038 \times 10^{-4}$$

$$\alpha = 2Y_0/d = 0.061 \text{ (Np/m)}$$

$$k_y = 2X_0/d = 6.28 \text{ (rad/m)}$$

$$\text{From Eq. (10-124): } \beta = \sqrt{\omega^2 \mu_d \epsilon_d - k_y^2} = 4.19 \text{ (rad/m)}$$

b) $f = 5 \times 10^8$ (Hz), $\lambda = c/f = 0.60$ (m), $k_0 d/2 = 0.0524$

$$A = 0.308, \quad R = 0.0785.$$

$$X_0 = 0.0785, \quad Y_0 = 1.901 \times 10^{-3}$$

$$\text{We obtain } \alpha = 0.380 \text{ (Np/m)}$$

$$\beta = 10.48 \text{ (rad/m)}$$

P.10-24 From Eq. (10-135):

$$\left(\frac{d}{2}\right) = -\frac{\epsilon_0}{\epsilon_d} \left(\frac{k_y d}{2}\right) \cot\left(\frac{k_y d}{2}\right) \quad (1)$$

Using the notations in problem P.10-23, we obtain two equations from (1) in P.10-23 and (2) above:

$$\begin{cases} X^2 + Y^2 = R^2 & (1) \\ Y = -AX \cot X & (2) \end{cases}$$

a) $f = 2 \times 10^8$ (Hz), $\lambda = 1.5$ (m)	b) $f = 5 \times 10^8$ (Hz), $\lambda = 0.60$ (m)
$A = 0.308$	$A = 0.308$
$R = 0.0314$	$R = 0.075$

There are no intersections for curves representing Eqs. (1) and (2); hence even TM modes do not exist at the given frequencies.

P.10-25 Use Eqs. (10-23d) and (10-23a):

$$E_y^0 = -\frac{j\beta}{h^2} \frac{\partial E_x^0}{\partial y}, \quad H_x^0 = \frac{j\omega\epsilon}{h^2} \frac{\partial E_x^0}{\partial y}$$

$$\bar{E}(y, z; t) = \alpha_0 [\bar{E}^0(y) e^{j(\omega t - \beta z)}]$$

$$\bar{H}(y, z; t) = \alpha_0 [\bar{H}^0(y) e^{j(\omega t - \beta z)}]$$

$|y| \leq d/2$:

$$E_x^0(y) = E_0 \cos k_y y \longrightarrow E_x(y, z; t) = E_0 \cos k_y y \cos(\omega t - \beta z)$$

$$E_y^0(y) = \frac{j\beta}{k_y} E_0 \sin k_y y \longrightarrow E_y(y, z; t) = -\frac{\beta}{k_y} E_0 \sin k_y y \sin(\omega t - \beta z)$$

$$H_x^0(y) = -\frac{j\omega\epsilon d}{k_y} E_0 \sin k_y y \longrightarrow H_x(y, z; t) = \frac{\omega\epsilon d}{k_y} E_0 \sin k_y y \sin(\omega t - \beta z)$$

$y \geq d/2$:

$$E_x^0(y) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \longrightarrow E_x(y, z; t) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \cos(\omega t - \beta z)$$

$$E_y^0(y) = -\frac{j\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \longrightarrow E_y(y, z; t) = \frac{\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \sin(\omega t - \beta z)$$

$$H_x^0(y) = -\frac{j\omega\epsilon}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \longrightarrow H_x(y, z; t) = -\frac{\omega\epsilon}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{-\alpha(y - \frac{d}{2})} \sin(\omega t - \beta z)$$

$y \leq -d/2$:

$$E_x^0(y) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \longrightarrow E_x(y, z; t) = E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \cos(\omega t - \beta z)$$

$$E_y^0(y) = \frac{j\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \longrightarrow E_y(y, z; t) = -\frac{\beta}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \sin(\omega t - \beta z)$$

$$H_x^0(y) = -\frac{j\omega\epsilon}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \longrightarrow H_x(y, z; t) = \frac{\omega\epsilon}{\alpha} E_0 \cos\left(\frac{k_y d}{2}\right) e^{\alpha(y + \frac{d}{2})} \sin(\omega t - \beta z)$$

P.10-26 a) From Table 10-2 on p. 485 it is seen that $f_{c_0} = 0$ for TE₁ mode, which is the dominant mode.

From Eq. (10-142):

$$\alpha = \frac{\mu_0}{\mu_d} k_y \tan \frac{k_y d}{2} \cong \frac{\mu_0 d}{2\mu_d} k_y^2, \text{ for } k_y d \ll 1.$$

Neglecting the α^2 term in Eq. (10-126):

$$\beta^2 - \omega^2 \mu_0 \epsilon_0 = \alpha^2 \approx 0 \longrightarrow \beta \cong \omega \sqrt{\mu_0 \epsilon_0} = k_0.$$

From Eq. (10-124): $k_y^2 = \omega^2 \mu_d \epsilon_d - \beta^2 \cong k_d^2 - k_0^2$

$$\therefore \alpha \cong \frac{\mu_0 d}{2\mu_d} (k_d^2 - k_0^2).$$

b) $d = 5 \times 10^{-3}$ (m), $\epsilon_d = 3\epsilon_0$, $\mu_d = \mu_0$, $f = 3 \times 10^8$ (Hz), $k_0 = 2\pi$.

$$\alpha = \frac{d}{2} k_0^2 (\epsilon_r - 1) = 0.197 \text{ (Np/m)}.$$

$$e^{-\alpha(y - \frac{d}{2})} = 0.368, \quad \alpha(y - \frac{d}{2}) = 1.$$

$$\longrightarrow (y - \frac{d}{2}) = 5.066 \text{ (m)}.$$

P.10-27 Use Eqs. (10-42b) and (10-42c):

$$H_y^0 = -\frac{j\beta}{k^2} \frac{\partial H_z^0}{\partial y}, \quad E_x^0 = -\frac{j\omega\mu_0}{k^2} \frac{\partial H_z^0}{\partial y}.$$

$$\bar{H}(y, z; t) = \mathcal{R}_e [\bar{H}^0(y) e^{j(\omega t - \beta z)}]$$

$$\bar{E}(y, z; t) = \mathcal{R}_e [\bar{E}^0(y) e^{j(\omega t - \beta z)}].$$

$|y| \leq d/2$:

$$H_z^0(y) = H_0 \cos k_y y \longrightarrow H_z(y, z; t) = H_0 \cos k_y y \cos(\omega t - \beta z)$$

$$H_y^0(y) = -\frac{j\beta}{k_y} H_0 \sin k_y y \longrightarrow H_y(y, z; t) = -\frac{j\beta}{k_y} H_0 \sin k_y y \sin(\omega t - \beta z)$$

$$E_x^0(y) = \frac{j\omega\mu_0}{k_y} H_0 \sin k_y y \longrightarrow E_x(y, z; t) = -\frac{\omega\mu_0}{k_y} H_0 \sin k_y y \sin(\omega t - \beta z)$$

$y \geq d/2$:

$$H_z^0(y) = H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \longrightarrow H_z(y, z; t) = H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \cos(\omega t - \beta z)$$

$$H_y^0(y) = -\frac{j\beta}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \longrightarrow H_y(y, z; t) = -\frac{j\beta}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \sin(\omega t - \beta z)$$

$$E_x^0(y) = \frac{j\omega\mu_0}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \longrightarrow E_x(y, z; t) = -\frac{\omega\mu_0}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{-\alpha(y - \frac{d}{2})} \sin(\omega t - \beta z)$$

$y \leq -d/2$:

$$H_z^0(y) = H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \longrightarrow H_z(y, z; t) = H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \cos(\omega t - \beta z)$$

$$H_y^0(y) = -\frac{j\beta}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \longrightarrow H_y(y, z; t) = -\frac{j\beta}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \sin(\omega t - \beta z)$$

$$E_x^0(y) = \frac{j\omega\mu_0}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \longrightarrow E_x(y, z; t) = -\frac{\omega\mu_0}{k_y} H_0 \cos(\frac{k_y d}{2}) e^{\alpha(y + \frac{d}{2})} \sin(\omega t - \beta z).$$

Setting $y = d/2$ in $E_x^{\circ}(y) = \frac{j\omega\mu_d}{k_y} H_o \sin k_y y$ and
in $E_x^{\circ}(y) = -\frac{j\omega\mu_o}{a} H_o \cos\left(\frac{k_y d}{2}\right) e^{-a(y-\frac{d}{2})}$

and equating, we obtain

$$E_x^{\circ}\left(\frac{d}{2}\right) = \frac{j\omega\mu_d}{k_y} H_o \sin\left(\frac{k_y d}{2}\right) = -\frac{j\omega\mu_o}{a} H_o \cos\left(\frac{k_y d}{2}\right)$$

$$\longrightarrow \frac{a}{k_y} = -\frac{\mu_o}{\mu_d} \cot\left(\frac{k_y d}{2}\right).$$

P.10-28 a) Odd TM and even TE modes are the propagating modes. Using $2d$ for d in the formulas in Table 10-2, p. 485, we have

$$f_{co} = \frac{n-1}{2d\sqrt{\mu_d\epsilon_d - \mu_o\epsilon_o}} \text{ for odd TM modes}$$

$$f_{ce} = \frac{n-\frac{1}{2}}{2d\sqrt{\mu_d\epsilon_d - \mu_o\epsilon_o}} \text{ for even TE modes.}$$

b) For odd TM modes — From Eqs. (10-127b and c):

$$|y| \leq d/2. \quad E_y^{\circ}(y) = -\frac{j\beta}{k_y} E_o \cos k_y y$$

$$H_x^{\circ}(y) = \frac{j\omega\epsilon_d}{k_y} E_o \cos k_y y.$$

Surf. current density on conductor $\vec{J}_s = \vec{a}_n \times \vec{H} \Big|_{y=0}$

$$\vec{J}_s = -\vec{a}_z H_x^{\circ}(0) = -\vec{a}_z \frac{j\omega\epsilon_d}{k_y} E_o.$$

Surf. charge density on conductor $\rho_s = \vec{a}_n \cdot \vec{D} \Big|_{y=0}$

$$\rho_s = \epsilon_d E^{\circ}(0) = -\frac{j\beta\epsilon_d}{k_y} E_o.$$

For even TE modes — From problem P.10-27:

$$|y| \leq d. \quad H_y^{\circ}(y) = \frac{j\beta}{k_y} H_o \sin k_y y$$

$$E_x^{\circ}(y) = \frac{j\omega\mu_d}{k_y} H_o \sin k_y y$$

$$H_z^{\circ}(y) = H_o \cos k_y y.$$

$$\therefore \vec{J}_s = \vec{a}_y \times [\vec{a}_y H_y^{\circ}(0) + \vec{a}_z H_z^{\circ}(0)] = \vec{a}_x H_o$$

$$\rho_s = \vec{a}_y \cdot \epsilon_d \vec{E}(0) = 0.$$

P.10-29 From Eq. (10-150): $f_{mnp} = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$.

$$f_{mnp} = 1.5 \times 10^{10} F(m,n,p), \quad F(m,n,p) = \sqrt{\left(\frac{m}{8}\right)^2 + \left(\frac{n}{6}\right)^2 + \left(\frac{p}{5}\right)^2}.$$

Lowest-order modes and resonant frequencies:

Modes	$F(m, n, p)$	$(f_c)_{\min}$ in (Hz)
TM_{110}	0.2083	3.125×10^9
TE_{101}	0.2358	3.538×10^9
TE_{011}	0.2603	3.905×10^9
TE_{111}, TM_{111}	0.2888	4.332×10^9
TM_{210}	0.3005	4.507×10^9
TE_{201}	0.3202	4.802×10^9
TM_{120}	0.3560	5.340×10^9
TE_{211}, TM_{211}	0.3609	5.414×10^9
TE_{021}	0.3887	5.821×10^9
TE_{121}, TM_{121}	0.4083	6.125×10^9

P.10-30 a) Since $d > a > b$, the lowest-order resonant mode is TE_{101} mode.

$$f_{101} = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{d^2}} = 4.802 \times 10^9 \text{ (Hz)}.$$

b) From Eq. (10-161):

$$Q_{101} = \frac{\pi f_{101} \mu_0 a b d (a^2 + d^2)}{R_s [2b(a^2 + d^2) + ad(a^2 + d^2)]} \quad \left(R_s = \sqrt{\frac{\pi f_{101} \mu_0}{\sigma}} \right)$$

$$= \frac{\sqrt{\pi f_{101} \mu_0 \sigma} a b d (a^2 + d^2)}{2b(a^2 + d^2) + ad(a^2 + d^2)} = 6869.$$

From Eqs. (10-156a) and (10-156b):

$$W_e = \frac{1}{4} \epsilon_0 \mu_0^2 a^3 b d f_{101}^2 H_0^2 = 0.07728 \times 10^{-12} \text{ (J)}$$

$$W_m = \frac{\mu_0}{16} a b d \left(\frac{a^2}{d^2} + 1 \right) H_0^2 = 0.07728 \times 10^{-12} \text{ (J)} = W_e.$$

P.10-31 a) $(f_{101})_{\epsilon_d} = \frac{u}{2} \sqrt{\frac{1}{a^2} + \frac{1}{d^2}} = \frac{1}{\sqrt{\epsilon_r}} (f_{101})_{\epsilon_0} = 3.037 \times 10^9 \text{ (Hz)}.$

b) $(Q_{101})_{\epsilon_d} = \frac{1}{(\epsilon_r)^{1/2}} (Q_{101})_{\epsilon_0} = 5462.$

c) $(W_e)_{\epsilon_d} = (W_e)_{\epsilon_0} = 0.07728 \times 10^{-12} \text{ (J)} = 0.07728 \text{ (pJ)}$
 $= (W_m)_{\epsilon_d}.$

P.10-32 a) $Q_{101} = \frac{\sqrt{\pi f_{101} \mu_0 \sigma} abd(a^2 + d^2)}{2b(a^2 + d^2) + ad(a^2 + d^2)}$

For $a = d = 1.8b$, $f_{101} = \frac{1}{2f \mu_0 \epsilon_0} \sqrt{\frac{1}{a^2} + \frac{1}{d^2}} = 1.179 \times 10^8 \left(\frac{1}{b}\right)$

$Q_{101} = 10.22 \sqrt{\epsilon_0} b$.

b) For $Q'_{101} = 1.20 Q_{101}$, $b' = 1.20^2 b = 1.44b$.

P.10-33 (I) From the field configurations in the cavity we see that the TM_{110} mode with respect to z is the same as the TE_{101} mode with respect to y . Thus, $(Q_{110})_{TM}$ can be obtained from $(Q_{101})_{TE}$ in Eq. (10-161) by changing b to d and d to b .

or, (II) Q for the TM_{110} mode can be derived from the field expressions in Eqs. (10-149a, d, and e) by setting $m=n=1$, and using Eq. (10-155).

$W = 2W_m = \frac{\mu_0}{8} \left(\frac{\omega^2 \epsilon_0^2}{h^2}\right) abd E_0^2$ at f_{110} .

$$\begin{aligned} P_L &= \oint \frac{1}{2} |\vec{J}_s|^2 R_s ds = \oint \frac{1}{2} |\vec{H}_t|^2 R_s ds \\ &= R_s \left\{ \int_0^d \int_0^b |H_y(z=0)|^2 dy dz + \int_0^d \int_0^a |H_x(y=0)|^2 dx dz \right. \\ &\quad \left. + \int_0^b \int_0^a [|H_x(z=0)|^2 + |H_y(z=0)|^2] dx dy \right\} \\ &= \frac{R_s}{2} \left(\frac{\omega^2 \epsilon_0^2}{h^2}\right) E_0^2 \left\{ \frac{1}{h^2} \left(\frac{\pi}{a}\right)^2 bd + \frac{1}{h^2} \left(\frac{\pi}{b}\right)^2 ad + \frac{1}{2} ab \right\}, \\ &\qquad\qquad\qquad h^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2. \end{aligned}$$

$Q_{110} = \frac{\omega_{110} W}{P_L} = \frac{\pi f_{110} \mu_0 abd(a^2 + b^2)}{R_s [2d(a^2 + b^2) + ab(a^2 + b^2)]}$, $R_s = \sqrt{\frac{\pi f_{110} \mu_0}{\sigma}}$

P.10-34 $C = \frac{\epsilon S}{d} = \frac{\epsilon \pi a^2}{d}$

$L = \frac{\mu h}{2\pi} \ln\left(\frac{b}{a}\right)$

a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\pi a \sqrt{\mu \epsilon} \sqrt{\frac{2h}{d} \ln\left(\frac{b}{a}\right)}}$

b) $\lambda = \frac{1}{f \sqrt{\mu \epsilon}} = \pi a \sqrt{\frac{2h}{d} \ln\left(\frac{b}{a}\right)}$.

Chapter 11

P.11-1 Maxwell's equations for simple media:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (2)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon} \quad (3)$$

$$\nabla \cdot \vec{H} = 0 \quad (4)$$

$$\begin{aligned} \text{a) } \nabla \times (1): \quad \nabla \times \nabla \times \vec{E} &= -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \\ &= -\mu \frac{\partial \vec{J}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \end{aligned} \quad (5)$$

$$\begin{aligned} \text{But } \nabla \times \nabla \times \vec{E} &= \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \\ &= \frac{1}{\epsilon} \nabla \rho - \nabla^2 \vec{E} \end{aligned} \quad (6)$$

Combining (5) and (6), we obtain

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\epsilon} \nabla \rho + \mu \frac{\partial \vec{J}}{\partial t}$$

$$\text{b) Similarly, we have } \nabla^2 \vec{H} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = -\nabla \times \vec{J}$$

P.11-2 Eq. (11-2): $\vec{E} = -\nabla V - j\omega \vec{A} = -\vec{a}_R E_R + \vec{a}_\theta E_\theta + \vec{a}_\phi E_\phi$

$$E_R = -\frac{\partial V}{\partial R} - j\omega A_R$$

$$E_\theta = -\frac{\partial V}{R \partial \theta} - j\omega A_\theta$$

$$E_\phi = -\frac{\partial V}{R \sin \theta \partial \phi} - j\omega A_\phi$$

The expressions of $A_R, A_\theta,$

and A_ϕ are given in Eqs.

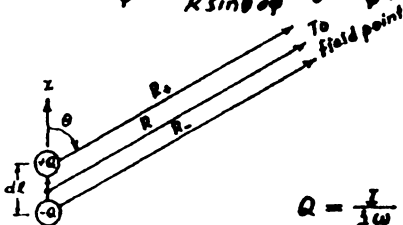
(11-14 a, b, and c).

$$V = \frac{Q}{4\pi\epsilon_0} \left[\frac{e^{-j\beta R_+}}{R_+} - \frac{e^{-j\beta R_-}}{R_-} \right]$$

$$R_+ \cong R - \frac{1}{2} dl \cos \theta$$

$$R_- \cong R + \frac{1}{2} dl \cos \theta$$

$$Q = \frac{I}{j\omega}, \quad (dl)^2 \ll R^2$$



$$V \cong \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2} \frac{1}{R^2} \left[\left(R + \frac{dl}{2} \cos \theta \right) e^{j\beta \left(dl \cos \theta \right) / 2} - \left(R - \frac{dl}{2} \cos \theta \right) e^{-j\beta \left(dl \cos \theta \right) / 2} \right]$$

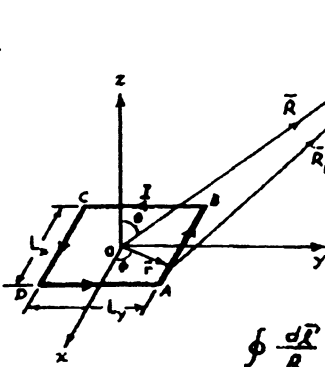
$$= \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2} \left[2jR \sin \left(\frac{\beta dl \cos \theta}{2} \right) + 2 \left(\frac{dl}{2} \cos \theta \right) \cos \left(\frac{\beta dl \cos \theta}{2} \right) \right]$$

$$\cong \frac{I e^{-j\beta R}}{4\pi\epsilon_0 j\omega R^2} \left[2jR \left(\frac{\beta dl \cos \theta}{2} \right) + dl \cos \theta \right]$$

$$= \frac{I dl \cos \theta}{4\pi R^2} \eta_0 \left(R + \frac{1}{j\beta} \right) e^{-j\beta R}$$

Using A_R, A_θ, A_ϕ , and V in E_R, E_θ , and E_ϕ , we obtain the same results as given in Eqs. (11-16a,b,c).

P.11-3



$$\begin{aligned} \bar{A} &= \frac{\mu_0 I}{4\pi} \oint \frac{e^{-j\beta R_1}}{R_1} d\bar{L}' \\ &= \frac{\mu_0 I}{4\pi} e^{-j\beta R} \oint \frac{e^{-j\beta(R-R_1)}}{R_1} d\bar{L}' \end{aligned}$$

$$\begin{aligned} \bar{A} &\approx \frac{\mu_0 I}{4\pi} e^{-j\beta R} \left[(1+j\beta R) \oint \frac{d\bar{L}'}{R_1} \right. \\ &\quad \left. - j\beta R \oint d\bar{L}' \right] \end{aligned}$$

$$\oint d\bar{L}' = 0.$$

$$\oint \frac{d\bar{L}'}{R_1} = \oint_{AB} \frac{d\bar{L}'}{R_1} + \oint_{CD} \frac{d\bar{L}'}{R_1} + \oint_{BC} \frac{d\bar{L}'}{R_1} + \oint_{DA} \frac{d\bar{L}'}{R_1}$$

$$R_1^2 = R^2 + r^2 - 2\bar{R} \cdot \bar{r}$$

$$\bar{R} = \bar{a}_x R \sin\theta \cos\phi + \bar{a}_y R \sin\theta \sin\phi + \bar{a}_z R \cos\theta$$

$$\bar{r} = \bar{a}_x x + \bar{a}_y \frac{L_y}{2}, \quad \bar{R} \cdot \bar{r} = R x \sin\theta \cos\phi + R \frac{L_y}{2} \sin\theta \sin\phi$$

$$\frac{1}{R_1} \approx \frac{1}{R} \left[1 + \frac{\bar{R} \cdot \bar{r}}{R^2} \right] = \frac{1}{R} \left(1 + \frac{x}{R} \sin\theta \cos\phi + \frac{L_y}{2R} \sin\theta \sin\phi \right)$$

$$\begin{aligned} \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \int_{AB} \frac{d\bar{L}'}{R_1} &= \bar{a}_x \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \frac{1}{R} \int_{-L_x/2}^{L_x/2} \left(1 + \frac{x}{R} \sin\theta \cos\phi + \frac{L_y}{2R} \sin\theta \sin\phi \right) dx \\ &= \bar{a}_x \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \frac{1}{R} \left(-L_x - \frac{L_x L_y}{2R} \sin\theta \sin\phi \right) \end{aligned}$$

In the same manner, we have

$$\frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \int_{CD} \frac{d\bar{L}'}{R_1} = \bar{a}_x \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \frac{1}{R} \left(L_x - \frac{L_x L_y}{2R} \sin\theta \sin\phi \right)$$

$$\therefore \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \int_{AB} \frac{d\bar{L}'}{R_1} = -\bar{a}_x \frac{\mu_0 I}{4\pi R^2} e^{-j\beta R} (1+j\beta R) L_x L_y \sin\theta \sin\phi$$

and

$$\frac{\mu_0 I}{4\pi} e^{-j\beta R} (1+j\beta R) \int_{BC} \frac{d\bar{L}'}{R_1} = \bar{a}_y \frac{\mu_0 I}{4\pi R^2} e^{-j\beta R} (1+j\beta R) L_x L_y \sin\theta \cos\phi$$

Let $m = I L_x L_y = IS$.

$$\bar{A} = \frac{\mu_0 m}{4\pi R^2} e^{-j\beta R} (1+j\beta R) \sin\theta (-\bar{a}_x \sin\phi + \bar{a}_y \cos\phi)$$

$$= \bar{a}_\phi \frac{\mu_0 m}{4\pi R^2} e^{-j\beta R} (1+j\beta R) \sin\theta.$$

$$c) \quad \bar{H} = \frac{1}{\mu_0} \nabla \times \bar{A} = \bar{a}_R H_R + \bar{a}_\theta H_\theta.$$

Expressions for H_R, H_θ , and E_ϕ same as those given in Eqs. (11-26a,b,c).

$$b) \quad \bar{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \bar{H} = \bar{a}_\phi E_\phi.$$

In the far zone, $\beta R \gg 1$, $1/(j\beta R)^2$ and $1/(j\beta R)^3$ terms can be neglected. We have the following instantaneous expressions; assuming $i(t) = I \cos \omega t$:

$$\bar{A}(R, \theta; t) = -\bar{a}_\phi \frac{\mu_0 m}{4\pi R} \beta \sin \theta \sin(\omega t - \beta R)$$

$$\bar{E}(R, \theta; t) = \bar{a}_\phi \frac{\omega \mu_0 m}{4\pi R} \beta \sin \theta \cos(\omega t - \beta R)$$

$$\bar{H}(R, \theta; t) = -\bar{a}_\theta \frac{m}{4\pi R} \beta^2 \sin \theta \cos(\omega t - \beta R).$$

P.11-4 Far-zone electric field of elemental electric dipole:

$$E_\theta(R) = j \frac{I_0 L}{4\pi} \left(\frac{e^{-j\beta R}}{R} \right) \eta_0 \beta \sin \theta \rightarrow E_\theta(R, t) = -\frac{I_0 \eta_0 \beta \sin \theta}{4\pi R} (L) \sin(\omega t - \beta R)$$

For the elemental magnetic dipole:

$$E_\phi(R) = \frac{\omega \mu_0 m}{4\pi} \left(\frac{e^{-j\beta R}}{R} \right) \beta \sin \theta \rightarrow E_\phi(R, t) = \frac{I_0 \eta_0 \beta \sin \theta}{4\pi R} \left(\frac{2\pi S}{\lambda} \right) \cos(\omega t - \beta R).$$

a) Thus,
$$\frac{E_\theta^2(R, t)}{\left(\frac{I_0 \eta_0 \beta \sin \theta}{4\pi R} \right)^2 (L)^2} + \frac{E_\phi^2(R, t)}{\left(\frac{I_0 \eta_0 \beta \sin \theta}{4\pi R} \right)^2 \left(\frac{2\pi S}{\lambda} \right)^2} = 1$$

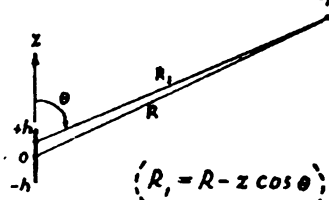
— — — Elliptic polarization.

b) Circular polarization if $L = 2\pi S/\lambda$.

P.11-5 a)
$$E_\theta = j \frac{I_0 \eta_0 \beta \sin \theta}{4\pi R} e^{-j\beta R} \int_{-h}^h \left(1 - \frac{|z|}{h} \right) e^{j\beta z \cos \theta} dz$$

$$= j \frac{I_0 \eta_0 \beta \sin \theta}{2\pi R} e^{-j\beta R} \int_0^h \left(1 - \frac{z}{h} \right) \cos(\beta z \cos \theta) dz$$

$$= \frac{j 60 I_0}{(\beta h) R} e^{-j\beta R} F(\theta)$$



$$(R_1 = R - z \cos \theta)$$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{j I_0}{(\beta h) 2\pi R} e^{-j\beta R} F(\theta).$$

$$F(\theta) = \frac{\sin \theta [1 - \cos(\beta h \cos \theta)]}{\cos^2 \theta}.$$

In case $\beta h \ll 1$, $\cos(\beta h \cos \theta) \approx 1 - \frac{1}{2} (\beta h \cos \theta)^2$, and

$$F(\theta) \approx \frac{1}{2} (\beta h)^2 \sin \theta.$$

$$\therefore E_\theta = \frac{j 60 I_0}{R} e^{-j\beta R} \left(\frac{1}{2} \beta h \sin \theta \right) = \frac{j 30 \beta h I_0}{R} e^{-j\beta R} \sin \theta$$

$$H_\phi = \frac{j I_0}{2\pi R} e^{-j\beta R} \left(\frac{1}{2} \beta h \sin \theta \right) = \frac{j \beta h I_0}{4\pi R} e^{-j\beta R} \sin \theta.$$

b)
$$W_r = \frac{1}{2} \int_0^{2\pi} \int_0^\pi E_\theta H_\phi^* R^2 \sin \theta d\theta d\phi = \frac{I_0^2}{2} \left[80\pi^2 \left(\frac{h}{\lambda} \right)^2 \right]$$

$$R_r = W_r / \left(\frac{1}{2} I_0^2 \right) = 20\pi^2 \left(\frac{2h}{\lambda} \right)^2.$$

c)
$$D = \frac{4\pi |E_{\max}|^2}{\int_0^{2\pi} \int_0^\pi |E_\theta(\theta)|^2 \sin \theta d\theta d\phi} = \frac{2}{\int_0^\pi \sin^3 \theta d\theta} = 1.5 \rightarrow 10 \log_{10} D = 1.76 \text{ (dB)}$$

$$\begin{aligned}
 \text{P.11-6} \quad 2h_e(\theta) &= \sin\theta \int_{-h}^h \sin\beta(h-|z|) e^{j\beta z \cos\theta} dz \\
 &= \frac{2[\cos(\beta h \cos\theta) - \cos\beta h]}{\beta \sin\theta}
 \end{aligned}$$

a) For half-wave dipole, $h = \lambda/4$, $\beta h = \pi/2$.

$$2h_e(\theta) = \frac{2\cos(\frac{\pi}{2}\cos\theta)}{\beta \sin\theta}$$

b) For maximum $2h_e(\theta)$, set $\frac{d}{d\theta} h_e(\theta) = 0 \rightarrow \theta = 90^\circ$ or 270° :

$$\text{Max. } 2h_e(\theta) = 2h_e(90^\circ \text{ or } 270^\circ) = \frac{\lambda}{\pi} = \left(\frac{2}{\pi}\right)\frac{\lambda}{2} = 0.637(2h)$$

$$\begin{aligned}
 \text{P.11-7} \quad W_r &= \oint \vec{\sigma}_{av} \cdot d\vec{s} = \frac{1}{2} \mathcal{R}_e \int_0^{2\pi} \int_0^\pi E_\phi H_\phi^* R^2 \sin\theta d\theta d\phi \\
 &= \frac{(IdL)^2}{16\pi} \beta^4 R^2 \eta_0 \mathcal{R}_e \left\{ \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] \left[-\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} - \frac{1}{(j\beta R)^3} \right] \right\} \\
 &\quad \cdot \int_0^\pi \sin^3\theta d\theta \\
 &= \frac{I^2}{2} \left[80\pi^2 \left(\frac{dL}{\lambda} \right)^4 \right], \text{ same as Eq. (11-37)}.
 \end{aligned}$$

9) P.11-8 From Eqs. (11-27a) and (11-27b):

$$E_\phi = \frac{\omega \mu_0 m}{4\pi} \left(\frac{e^{-j\beta R}}{R} \right) \beta \sin\theta$$

$$H_\theta = -\frac{\omega \mu_0 m}{4\pi \eta_0} \left(\frac{e^{-j\beta R}}{R} \right) \beta \sin\theta$$

$$W_r = \frac{1}{2} \mathcal{R}_e \int_0^{2\pi} \int_0^\pi (-E_\phi H_\theta^*) R^2 \sin\theta d\theta d\phi = \left(\frac{I^2}{2} \right) 320\pi^4 \left(\frac{S}{\lambda^2} \right)^2$$

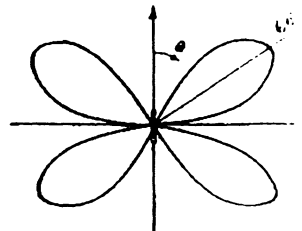
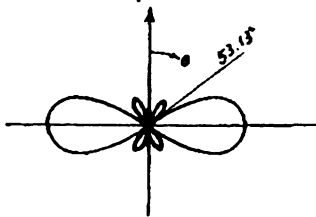
$$\therefore R_r = \frac{W_r}{(I^2/2)} = 320\pi^4 \left(\frac{S}{\lambda^2} \right)^2$$

a) Circular loop of radius b : $R_r = 320\pi^4 \left(\frac{b}{\lambda} \right)^4 = 20\pi^2 \left(\frac{2\pi b}{\lambda} \right)^4$

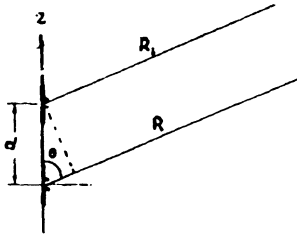
b) Rectangular loop of side L_x and L_y : $R_r = 320\pi^4 \left(\frac{L_x L_y}{\lambda^2} \right)^2$

$$\text{P.11-9} \quad F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos\beta h}{\sin\theta}$$

$$\text{a) } 2h = 1.25\lambda, |F(\theta)| = \left| \frac{\cos(1.25\pi \cos\theta) - \cos(1.25\pi)}{\sin\theta} \right| \quad \text{b) } 2h = 2\lambda, |F(\theta)| = \left| \frac{\cos(2\pi \cos\theta) - 1}{\sin\theta} \right|$$



P.11-10



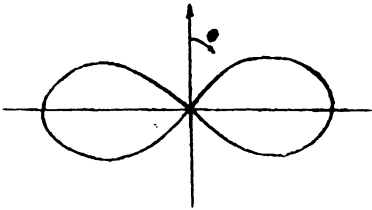
$$a) E_{\theta} = E_{\theta 1} + E_{\theta 2} = \frac{j2Ih}{4\pi R} \eta_0 \beta \sin \theta e^{j\beta R} \cdot (1 + e^{j\beta d \cos \theta})$$

$$= \frac{j60Ih}{R} 2\beta e^{-j\beta(R - \frac{d}{2} \cos \theta)} F(\theta)$$

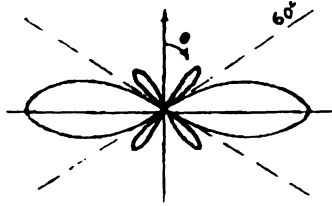
where

$$F(\theta) = \sin \theta \cos \left(\frac{\beta d}{2} \cos \theta \right)$$

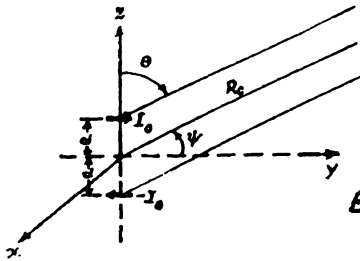
b) $d = \frac{\lambda}{2}, |F(\theta)| = |\sin \theta \cos(\frac{\pi}{2} \cos \theta)|$



c) $d = \lambda, |F(\theta)| = |\sin \theta \cos(\pi \cos \theta)|$



P.11-11



From Eq. (11-19b):

$$E_{\psi_1} = \frac{jI_0 dl \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 - d \cos \theta)} \sin \psi$$

$$E_{\psi_2} = -\frac{jI_0 dl \eta_0 \beta}{4\pi R_0} e^{-j\beta(R_0 + d \cos \theta)} \sin \psi$$

$$E_{\psi} = E_{\psi_1} + E_{\psi_2}$$

$$= \frac{jI_0 dl \eta_0 \beta}{4\pi R_0} e^{-j\beta R_0} 2 \sin(\beta d \cos \theta) \sin \psi$$

$$E_{\psi} = j \frac{I_0 dl}{2\pi} \left(\frac{e^{-j\beta R_0}}{R_0} \right) \eta_0 \beta \sin(\beta d \cos \theta) \sqrt{1 - \sin^2 \phi \sin^2 \theta}$$

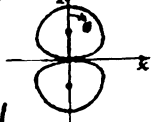
$$\bar{E}_{\psi} = \bar{a}_{\theta} E_{\theta} + \bar{a}_{\phi} E_{\phi} = -\frac{E_{\psi}}{\sqrt{1 - \sin^2 \phi \sin^2 \theta}} (\bar{a}_{\theta} \cos \theta \sin \phi + \bar{a}_{\phi} \cos \phi)$$

a) In the xy-plane, $\theta = 90^\circ, F_{xy}(\theta, \phi) = 0$.

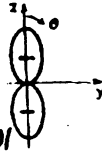
b) In the xz-plane, $\phi = 0^\circ, E_{\psi} = -E_{\phi}, |F_{xz}(\theta)| = |\sin(\beta d \cos \theta)|$

c) In the yz-plane, $\phi = 90^\circ, E_{\psi} = -E_{\theta}, |F_{yz}(\theta)| = |\cos \theta \sin(\beta d \cos \theta)|$.

d) $d = \lambda/4, \beta d = \pi/2$:



$$|F_{xz}| = |\sin(\frac{\pi}{2} \cos \theta)|$$



$$|F_{yz}| = |\cos \theta \sin(\frac{\pi}{2} \cos \theta)|$$

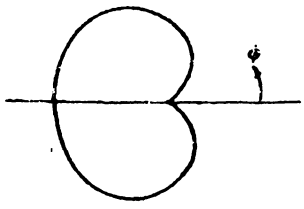
P.11-12 From Eq. (11-57) $E = \frac{3}{2} E_0 \cos^2 \frac{\psi}{2}$, where

$$\psi = k d \sin \theta \cos \phi$$

In the H -plane of $\alpha = \phi = \pi/2$, $\psi = \pi/2$, $F(\frac{\pi}{2}, \phi) = 1$.

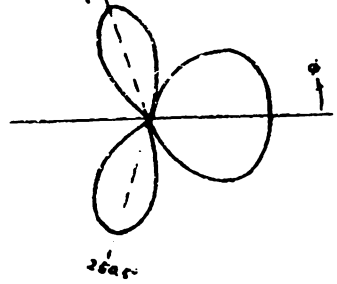
a) $d = \frac{\lambda}{4}$, $\frac{\psi}{2} = \frac{\pi}{2}$

$$|f(\phi)| = \left| \cos \left[\frac{\pi}{4} (1 + \cos \phi) \right] \right|$$



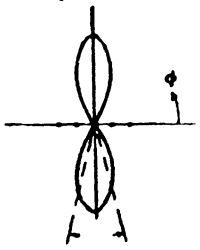
b) $d = \frac{3\lambda}{4}$, $\frac{\psi}{2} = \frac{\pi}{4}$

$$|f(\phi)| = \left| \cos \left(\frac{3\pi}{4} \cos \phi + \frac{\pi}{4} \right) \right|$$



P.11-13 a) Relative excitation amplitudes: 1:4:6:4:1.

b) Array factor: $|A(\phi)| = \left| \cos \left(\frac{\pi}{2} \cos \phi \right) \right|^4$



c) $\cos \left(\frac{\pi}{2} \cos \phi \right) = (\sqrt{2})^{-1/4}$

$$\rightarrow \phi = 74.86^\circ$$

Half-power beamwidth

$$= 2(90^\circ - 74.86^\circ)$$

$$= 30.28^\circ$$

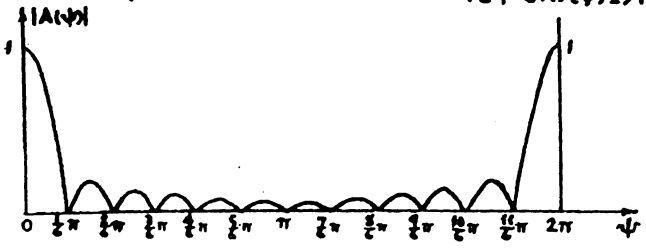
For uniform array, from Eq. (11-62):

$$\frac{1}{5} \left| \frac{\sin \left(\frac{5\pi}{2} \cos \phi \right)}{\sin \left(\frac{\pi}{2} \cos \phi \right)} \right| = \frac{1}{\sqrt{2}} \rightarrow \phi = 79.61^\circ$$

Half-power beamwidth for 5-element uniform array

$$\text{with } \lambda/2 \text{ spacing} = 2(90^\circ - 79.61^\circ) = 20.78^\circ$$

P.11-14 a) From Eq. (11-62) for $N=12$: $|A(\psi)| = \frac{1}{12} \left| \frac{\sin 6\psi}{\sin(\psi/2)} \right|$



b) Broadside Operation. $\psi = \beta d \cos \phi$.

$$|A(\psi)| = \frac{1}{N} \left| \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right| \cong \left| \frac{\sin X}{X} \right| \text{ for } \psi \ll 1,$$

where $X = N\psi/2$.

At half-power points: $\left| \frac{\sin X}{X} \right| = \frac{1}{\sqrt{2}} \rightarrow X = 1.391$

(For both broadside & endfire operations)

For broadside operation, the half-power beamwidth

is $(2\Delta\phi)_{1/2} = 0.886 \left(\frac{\lambda}{Nd} \right)$ (rad.)

$= 50.75 \left(\frac{\lambda}{Nd} \right)$ (deg.)

For $N=12$, $(2\Delta\phi)_{1/2} = 4.23 \left(\frac{\lambda}{d} \right)$ (deg.)

From Eq. (11-65): $(2\Delta\phi)_0 = 9.55 \left(\frac{\lambda}{d} \right)$ (deg.)

c) Endfire Operation. $\psi = \beta d (\cos \phi - 1)$

$(2\Delta\phi)_{1/2} = 1.882 \sqrt{\frac{\lambda}{Nd}}$ (rad.) $= 107.8 \sqrt{\frac{\lambda}{Nd}}$ (deg.)

For $N=12$, $(2\Delta\phi)_{1/2} = 31.13 \sqrt{\frac{\lambda}{d}}$ (deg.)

From Eq. (11-66): $(2\Delta\phi)_0 = 46.78 \sqrt{\frac{\lambda}{d}}$ (deg.)

P.11-15

$|A(\psi)| = \frac{1}{N} \left| \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right| \cong \left| \frac{\sin X}{X} \right|$, where $X = \frac{N\psi}{2}$.

Assume broadside operation: $\psi = \beta d \cos \theta$.



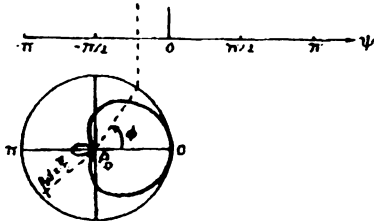
$$D = \frac{4\pi |A(\psi)_{\max}|^2}{\int_0^{2\pi} \int_0^{\pi} \left| \frac{\sin X}{X} \right|^2 \sin \theta \, d\theta \, d\phi}$$

$|A(\psi)_{\max}| = 1$,

$$\int_0^{\pi} \left| \frac{\sin X}{X} \right|^2 \sin \theta \, d\theta \cong \frac{4}{N\beta d} \int_0^{\infty} \left| \frac{\sin X}{X} \right|^2 dX = \frac{4}{N\beta d} \left(\frac{\pi}{2} \right) = \frac{\lambda}{Nd}$$

$\therefore D = \frac{2Nd}{\lambda} \cong \frac{2L}{\lambda}$, where $L = \text{array length}$.

P.11-16 Construction follows the steps outlined on pp. 525-526.



P_0 is at $\xi = -\frac{\pi}{2}$.

Radius of circle is $\beta d = \frac{2\pi}{\lambda} \left(\frac{\lambda}{4} \right) = \frac{\pi}{2}$.

P.11-17 From Eq. (11-43):

$$E_{\theta} = \frac{j60 I_m N_1 N_2}{R} e^{-j\beta R} \left[\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] |A_x(\psi_x) A_y(\psi_y)|$$

Where

$$|A_x| = \frac{1}{N_1} \left| \frac{\sin(N_1 \psi_x / 2)}{\sin(\psi_x / 2)} \right|, \quad \psi_x = \frac{\beta d_1}{2} \sin \theta \cos \phi;$$

$$|A_y| = \frac{1}{N_2} \left| \frac{\sin(N_2 \psi_y / 2)}{\sin(\psi_y / 2)} \right|, \quad \psi_y = \frac{\beta d_2}{2} \sin \theta \cos \phi.$$

$$|F(\theta, \phi)| = \frac{1}{N_1 N_2} \left[\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] \left| \frac{\sin(\frac{N_1 \psi_x}{2}) \sin(\frac{N_2 \psi_y}{2})}{\sin(\frac{\psi_x}{2}) \sin(\frac{\psi_y}{2})} \right|$$

P.11-18 From Eq. (11-77): $P_L = A_e \rho_{av}$. ①

Consider an elemental electric (Hertzian) dipole of length dL in the field of an incident plane wave with an electric intensity E_i .

$$\rho_{av} = \frac{|E_i|^2}{2\eta_0} \quad \text{②}$$

Maximum power is absorbed by the load if $Z_L = Z_0^*$.

$$P_L = \frac{1}{2} |I|^2 R = \frac{1}{2} \left(\frac{E_i dL}{Z_0 + Z_0^*} \right)^2 R = \frac{(E_i dL)^2}{8R} \quad \text{③}$$

Combining ①, ②, and ③, we have

$$\left. \begin{aligned} A_e &= \frac{\eta_0}{4R} (dL)^2 = \frac{30\pi}{R} (dL)^2 \\ \text{From Eq. (11-38): } R &= 80\pi^2 \left(\frac{dL}{\lambda} \right)^2 \end{aligned} \right\} \longrightarrow A_e = \frac{3}{8\pi} \lambda^2$$

$$\text{From p.511, } D = G_0 \left(\frac{\pi}{2} \cdot \phi \right) = \frac{1}{2} \longrightarrow \frac{A_e}{D} = \frac{\lambda^2}{4\pi}$$

P.11-19 From Eq. (11-89): $\frac{P_L}{P_T} = \left(\frac{\lambda}{4\pi r} \right)^2 G_{01} G_{02}$.

a) For half-wave dipoles: $G_{01} = G_{02} = 1.64$

$$f = 3 \times 10^8 \text{ (Hz)}, \quad \lambda = c/f = 1 \text{ (m)}, \quad r = 1.5 \times 10^3 \text{ (m)}$$

$$P_L = 7.57 \times 10^{-9} P_T = 7.57 \times 10^{-7} \text{ (W)} = 0.757 \text{ (}\mu\text{W)}$$

b) For Hertzian dipoles: $G_{01} = G_{02} = 1.5$

$$P_L = 0.633 \text{ (}\mu\text{W)}$$

P.11-20 Let P_T = Power intercepted by the target.

$$a) \quad \frac{P_T}{P_T} = \frac{A_T G_D}{4\pi r^2}, \quad \frac{P_L}{P_T} = \frac{A_D G_T}{4\pi r^2}; \quad G_T = \frac{4\pi}{\lambda^2}, \quad A_D = \frac{\lambda^2}{4\pi} G_D$$

$$\therefore \frac{P_t}{P_i} = \left(\frac{P_r}{P_t}\right) \left(\frac{P_r}{P_t}\right) = \frac{A_T^2 G_D^2}{(4\pi r^2)^2}$$

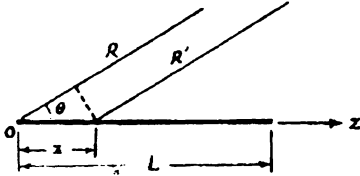
b) Incident power density at the target, $\sigma_i = \frac{P_i}{4\pi r^2} G_D$.

Power scattered by the target in the direction of the antenna, $P_{sc} = \sigma_i A_T G_T = \sigma_i \frac{4\pi}{\lambda^2} A_T^2$

$$\therefore S_r = \frac{P_{sc}}{\sigma_i} = \frac{4\pi}{\lambda^2} A_T^2$$

From the result of part a): $\frac{P_t}{P_i} = \frac{S_r G_D^2 \lambda^2}{(4\pi)^2 r^4}$

P.11-21



$$I(z) = I_0 e^{-j\beta z}$$

$$a) \bar{A} = \bar{a}_z \frac{\mu}{4\pi} \int_0^L \frac{I_0 e^{-j\beta z} e^{-j\beta R'}}{R'} dz$$

In the far-zone, $R' \approx R - z \cos \theta$.

$$\bar{A}(R, \theta) = \bar{a}_z \frac{\mu I_0 e^{-j\beta R}}{4\pi R} \int_0^L e^{-j\beta z(1-\cos\theta)} dz$$

$$= \bar{a}_z \frac{\mu I_0}{2\pi R \beta} e^{-j\beta R} e^{-j\beta \frac{L}{2}(1-\cos\theta)} F(\theta)$$

$$\text{where } F(\theta) = \frac{\sin\left[\frac{\beta L}{2}(1-\cos\theta)\right]}{1-\cos\theta}$$

b) $A_R = A_z \cos \theta$, $A_\theta = -A_z \sin \theta$, $A_\phi = 0$.

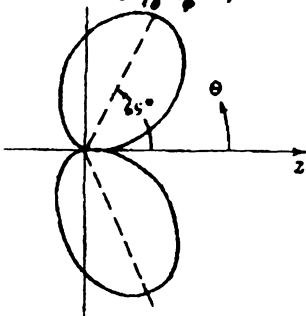
$$\bar{H} = \frac{1}{\mu} \nabla \times \bar{A} = \bar{a}_\phi \frac{1}{\mu R} \left[\frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

In the far-zone, $\frac{1}{R} \frac{\partial A_R}{\partial \theta} \propto \frac{1}{R^2} \rightarrow \bar{H} = \bar{a}_\phi \frac{1}{\mu R} \frac{\partial}{\partial R} (R A_\theta)$

$$\bar{H}(R, \theta) = \bar{a}_\phi \frac{j I_0}{2\pi R} e^{-j\beta[R+L(1-\cos\theta)/2]} F(\theta) \sin \theta$$

$$\bar{E}(R, \theta) = \bar{a}_\theta \eta_0 H_\phi(R, \theta)$$

c)



Radiation pattern for $L = \lambda/2$:

Plot of $|F(\theta) \sin \theta|$.

P.11-22 From Eq. (11-97b):

$$F(\theta, \phi) = \iint_{\text{aper.}} f(x', y') e^{j\beta \sin\theta (x' \cos\phi + y' \sin\phi)} dx' dy'$$

a) In the xz -plane, $\phi = 0^\circ$:

$$\begin{aligned} F_{xz}(\theta) &= b \int_{-a/2}^{a/2} f(x') e^{j\beta x' \sin\theta} dx' \\ &= 2b \int_0^{a/2} \left(1 - \frac{2}{a} x'\right) \cos(\beta x' \sin\theta) dx' \\ &= ab \frac{1 - \cos\left(\frac{\beta a}{2} \sin\theta\right)}{\left(\frac{\beta a}{2} \sin\theta\right)^2} \quad \text{Let } \psi = \frac{\beta a}{2} \sin\theta = \frac{\pi a}{\lambda} \sin\theta \end{aligned}$$

$$F_{xz}(\theta) = \frac{ab}{2} \left[\frac{\sin(\psi/2)}{(\psi/2)} \right]^2$$

b) Set $\left[\frac{\sin(\psi/2)}{(\psi/2)} \right]^2 = \frac{1}{2} \longrightarrow \frac{\psi}{2} = 1.005$.

Half-power beamwidth $(2\Delta\theta)_{1/2} = 2 \sin^{-1}(0.640 \frac{\lambda}{a})$.

For $\lambda/a \ll 1$, $(2\Delta\theta)_{1/2} \approx 1.280 \frac{\lambda}{a}$ (rad)
 $= 73.3 \frac{\lambda}{a}$ (deg).

c) Set $\frac{\psi}{2} = \pi \longrightarrow \theta_{n1} = \sin^{-1}\left(\frac{2\lambda}{a}\right) \approx \frac{2\lambda}{a}$ (rad)
 $= 114.6 \frac{\lambda}{a}$ (deg).

d) First-sidelobe level: $L_1 = -20 \log_{10} \left(\frac{1}{3\pi/2} \right)^2 = 26.9$ (dB).

	Uniform Distr.	Triangular Distr.
Pattern function	$ab \left(\frac{\sin\psi}{\psi} \right)$	$\frac{ab}{2} \left(\frac{\sin\frac{\psi}{2}}{\frac{\psi}{2}} \right)^2$
Half-power beamwidth	$50 \frac{\lambda}{a}$ (deg)	$73.3 \frac{\lambda}{a}$ (deg)
Location of first null	$57.3 \frac{\lambda}{a}$ (deg)	$114.6 \frac{\lambda}{a}$ (deg)
First-sidelobe level	13.3 (dB)	26.9 (dB)

P.11-23 a) In the xz -plane, $\phi = 0^\circ$:

$$\begin{aligned} F_{xz}(\theta) &= 2b \int_0^{a/2} \cos\left(\frac{\pi x'}{a}\right) \cos(\beta x' \sin\theta) dx' \\ &= \frac{2ab}{\pi} \left[\frac{(\pi/2)^2 \cos\psi}{(\pi/2)^2 - \psi^2} \right], \quad \psi = \frac{\beta a}{2} \sin\theta = \frac{\pi a}{\lambda} \sin\theta \end{aligned}$$